Auditors' Liability, Investments and Capital Markets: An Unintended Consequence of the Sarbanes-Oxley Act

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Abstract: To restore investors' confidence in the reliability of corporate financial disclosures, the Sarbanes-Oxley Act of 2002 mandated stricter regulations and arguably increased auditors’ liability. In this paper, we analyze the effects of increased auditor liability on the audit failure rate, the cost of capital, and the level of new investment. We focus on a setting in which with imperfect auditing, a firm has better information than investors about its prospects and seeks to raise capital in a lemons market for new investments. The equilibrium analysis derives corporate reporting and investing choices by the firm, attestation opinions by the auditor, and valuation by rational investors. Three predictions emerge that are empirically testable: While increasing auditor liability decreases the audit failure rate and decreases the cost of capital for new projects, it decreases the level of new profitable investments.

Keywords: Auditing; audit liability; information asymmetry; capital market.
1 Introduction

A number of highly publicized controllership failures of public companies and their auditors have led to the Sarbanes-Oxley Act of 2002 (hereafter SOX). SOX aimed to improve the reliability and usefulness of corporate financial disclosures. Section 404 of SOX required that auditors attest not only to the fairness and representational faithfulness of financial statements, but also the effectiveness of the internal controls system. In case of an audit failure, the auditor may be subject to criminal prosecution, and may face greater legal liability.¹ More recently, in the aftermath of the Satyam controllership failure in India, once again, questions about the appropriateness of auditors’ legal responsibility and their liability exposure have surfaced in the business press as well as among the regulators in India, US, Europe, and elsewhere.²

Capital market participants and regulators, however, have been concerned about raising legal liability for auditors. In the U.S.A., the Committee on Capital Markets Regulation recommends that “Congress should explore protecting audit firms against catastrophic loss through the provision of caps or safe harbors.”³ Treasury Secretary Henry Paulson and SEC chief accountant Conrad Hewitt both concurred with the views that the Big Four accounting firms are exposed to potentially large legal liability and that they should be given legal protection against damage claims.⁴⁵⁶ Meanwhile, European Union internal market commissioner Charlie McCreevy has already come out in favor of a fixed cap on the auditors’ liability claims.⁷ The U.K. Department of Trade has proposed the Companies (Audit, Investigations and Community Enterprise) Bill to cap auditors’ liability when they audit publicly traded companies, arguing that the consequence of growing litigation risk may result in more economic damage than it prevents. As part of the proposal, auditors will be able to negotiate proportionate liability by contracting with their clients subject to shareholders’ approval.⁸

¹We should note that SOX did not explicitly raise auditor’s liability. However, early evidence suggests audit liability is higher after SOX. For example, Rashkover and Winter (2005) argue that civil monetary penalties have increased because the SOX empowers federal courts and the SEC to impose equitable remedies for violations of federal securities laws. Ghosh and Pawlewicz (2008) document that audit fees after SOX are higher partially because of higher legal liability due to an audit failure. Also see the references cited therein.
²http://www.atimes.com/atimes/South_Asia/KA10Df02.html
³http://www.capmktsgreg.org/research.html
⁷http://ec.europa.eu/commission_barroso/mccreevy/allspeeches_en.htm
⁸For example, see WSJ, April 13, 2004 “In Closely Watched Move, U.K. May Cap Auditor Liability: Even as the corporate-
It is well established in the literature that in the presence of information asymmetry between firms and investors, efficient firms may forgo profitable investment opportunities (see Myers and Majluf (1984)). To reduce the cost of information asymmetry, firms may convey the quality of investment projects by issuing signals, such as financial reports, to investors. Such disclosures are most effective in reducing information asymmetry pertaining to hard information; that is, information that is verifiable once disclosed.\(^9\) This, in turn, creates demand for auditor attestation regarding the reliability of financial reports. Imposing higher auditor liability will not only change the auditors’ attestation strategy, but also affect investors’ ex-post assessment of the financial report. As a result, the firms’ ex ante optimal investment decisions are also affected.

To analyze consequences of increased auditor liability, we focus on a setting where the auditor’s role is to alleviate the information asymmetry between investors and firms. Specifically, our model combines and extends the “lemons” investment model of Myers and Majluf (1984) and the imperfect auditing model in Shibano (1990). We introduce an auditor to Myers and Majluf (1984) and extend Shibano (1990) hidden information audit model by incorporating investors with rational expectations, thus endogenizing the valuation of the firm. In the Myers and Majluf model, a firm has an investment opportunity with a positive net present value. It must issue equity in order to raise the investment funds needed; otherwise, the opportunity will be lost. The firm invests in the new project if its cost of capital is low enough. When a firm has private information regarding the value of its existing assets and liabilities, investors’ assessment of firm value may be lower than the firm’s assessment. In such a case, the firm’s cost of capital includes an asymmetric information premium that may be so high that the firm underinvests; that is, it foregoes a profitable investment project.

Mandatory audits of corporate financial disclosures serve to reduce the informational asymmetry, but potential audit failures may lessen such effect. The role of the auditor is to provide attestation to corporate financial disclosures about existing assets and liabilities.\(^{10}\) When the auditor’s attestation is perfect, the scandal body count rises in Europe, auditors, led by the Big Four accounting firms, are trying to limit the amount of money they can be forced to pay out when a company blows up on their watch. And they may be on the verge of a significant victory. Capping auditor liability has been a long-held objective of the accounting industry around the world, which has seen some success in the past in countries such as Germany. But the issue became a tough sell with legislators and the public in the wake of scandals in which auditors were cast in a harsh light, from Enron in the U.S. to, most recently, Parmalat in Italy.\(^9\)


\(^{10}\)Other roles of auditing include monitoring managerial contracts (e.g., Watts and Zimmerman (1983) and Antle and Demski (1991)), risk sharing with managers or investors (as in Antle (1982)), regulatory compliance, and informational needs of lenders.
investors’ confidence in the reliability and accuracy of information reported is assured, thereby reducing the information asymmetry and the lemons premium. However, the auditor’s attestation is imperfect; the auditor must make a judgment call based on limited information and therefore the auditor’s opinion is subject to unintentional errors. The auditor may unknowingly attest to financial statements that undervalue or overvalue existing equity after observing imperfect audit evidence.

An increase in the liability for audit failures may reduce the incentive for new profitable investments. Since the auditor’s attestation is subject to possible audit failures, an increase in the liability for audit failures induces the auditor to become more conservative in his interpretation of financial disclosures. As a result, the firm needs to choose its corporate disclosure strategy and an investment decision based on its anticipation of both investors’ valuation and a possible audit failure. When a firm has private information about its existing equity, the chances of receiving understated financial statement increases. As long as the new shareholders cannot infer the true value from the understated reports, the firm’s valuation will be depressed because of the lemons problem, thereby increasing the firm’s cost of capital and making new investments less likely. In sum, our results identify an economic consequence of SOX that has not been asserted in the literature: the increase in the audit liability after SOX has the countervailing effects of decreasing both audit failures and new investments.

Our study suggests several implications of SOX for investors, auditors, and policy makers. In the presence of information asymmetry between the firm and investors, an increase in audit liability attributed to SOX results in greater investors’ protection and confidence and, as such, encourages investment. However, if the information asymmetry regarding the value of existing equity is large, higher legal liability may exacerbate a lemons problem and result in reduced level of new investments. This countervailing effect, nevertheless, can be mitigated by improving the informativeness of financial disclosures, thereby reducing the cost of information asymmetry.

Our paper is related to several papers in the extant literature analyzing the impact of audit liability on the firm’s investment decisions (these include Antle and Nalebuff (1991), Schwartz (1997), and Lu and Sapra (2009)). These papers model a capital market valuation rule, an auditor’s attestation strategy and a firm’s reporting strategy and its investment decision. In these papers, since there is no information asymmetry, (as in Melumad and Thoman (1990)).
the auditor’s role is not to verify a firm’s strategic report, but rather to identify the nature of the firm’s operation activities. Naturally, the equilibrium investment level depends on a firm’s audit fees/liability ratio. A low fee-to-liability ratio makes an auditor more conservative, which, in turn, reduces audit quality and therefore the degree of investment inefficiency. Along the same line of reasoning, Laux and Newman (2009) analyze the effect of audit liability on client acceptance. In their setting, the source of potential over or under investment is the auditor’s moral hazard problem with respect to the client evaluation task.

In the presence of information asymmetry, the relation between audit conservatism and the firm’s reporting strategies are more involved. On the one hand, the firm may have various reporting strategies depending on the auditor’s attestation strategy and the investor’s valuation. We narrow nine possible reporting strategies down to two unique equilibriums (see Table 2). On the other hand, whether the auditor’s attestation strategy can effectively mitigate information asymmetry depends on the level of audit liability and on the firm’s reporting strategies. An increase in the liability for audit failures induces the auditor to become more conservative in his interpretation of financial disclosures and in turn, to issue an unfavorable report more frequently. The firm’s reporting strategies and the level of audit liability jointly determine the equilibrium strategies in three different regimes (ineffective, detection and deterrence) and the extent of investment inefficiency in our model (see Table 1). Focusing on the auditor’s role of mitigating the lemon problem, our study contributes to Laux and Newman (2009) and Lu and Sapra (2009) by showing that in presence of information asymmetry, a mixed reporting strategy may emerge in equilibrium when the audit adjustment cost is non-trivial and the auditor cannot effectively identify the firm’s true type, which subsequently leads to investment inefficiency.

Our study is also related to the finance, economics and accounting literature studying the role of various mechanisms in alleviating the lemon problem. These mechanisms include reputation, licensing, signaling by retained ownership, minimum quality standards, entry restriction, optimal contracting, and choice of capital structure (see Akerlof (1970), Datar, Feltham and Hughes (1991), Leland and Pyle (1977), Leland (1979, 1980), Myers and Majluf (1984), Dybvig and Zender (1991)). We assume that there are practical limitations that prevent achieving the optimal contracts between the firms and new investors described in Dybvig and Zender (1991).11 Specifically, because the audit technology is noisy, the firm may not always

11Dybvig and Zender explain how limitations induced by the inability to precommit, by restrictions on indentured servitude, and by corporate governance rules can preclude complete solution to the underinvestment problem through contracting. For instance,
be able to signal its type via auditors’ attestation. This analytical approach is not foreign to the literature on how a middleman (an auditor in our context) helps guarantee and monitor product quality, thereby mitigating information asymmetry between buyers and suppliers (see Biglaiser (1993), Biglaiser and Friedman (1994), and Albano and Lizzeri (2001)). We contribute to this literature by showing that while the auditor’s noisy attestation reduces the cost of asymmetric information, his conservative behavior may distort the firm’s incentives and affect the investors’ valuation.

Interestingly, the extant literature in auditing suggests that increasing auditor’s liability may have positive effects on audit quality and investors’ confidence. Two notable studies are Schwartz (1997) and Newman et al. (2005) who specifically look at the interaction between audit penalties, audit quality and investors’ investments. Their results suggest that an increase in auditors’ liability may give rise to overinvestment. Schwartz (1997) argues that the auditor’s liability provides insurance for investors against bad states of nature, and this, in turn, may lead to overinvestment. She also establishes that both the socially optimal investment and the socially optimal audit effort can be induced by a strict liability rule with a damage measure that is independent of the actual investment. Newman et al. (2005) shows that markets with greater auditor penalties for audit failure and greater insider penalties may lead to larger total investment levels, a higher proportion of the firms held by outsiders and higher investment returns. Along the same line of reasoning, Laux and Newman (2009) analyze the effect of audit liability on client acceptance rather than capital investment.

Early theoretical studies in the accounting literature tended to focus on studying the impact of audit quality on firm investments, showing that because higher audit quality provides better information to investors, the legal regime that induces the highest audit quality also generates the most efficient investment. For example, Dye (1993) suggests that equilibrium audit fees depend on both the informational value of the audit and on the auditor’s wealth. Melumad and Thoman (1990) demonstrate that the presence of a court system, increasing the exogenously specified damages, may lead to a Pareto improvement. Smith and Tidrick (1998) examine a model of auditing and settlement under the U.S. and British systems and showed that more auditing can be a function of the unit cost of audit. Thoman and Zhang (1999) point out that audit effort increases with the size of the damage award on audit effort, but may decrease with rigor of the

if the manager’s tenure is likely to end prior to the revelation of type and if penalties on the manager beyond his tenure are not feasible, optimal contracts are not implementable.
auditing standards. Pae and Yoo (2001) indicate that an increase in auditors’ legal liability may result in underinvestment in the internal control system and overinvestment in auditors’ effort.

In recent years, theoretical research on auditing has focused on characterizing the impact of specific institutional penalty alternatives on auditors’ effort, such as proportionate liability rules supported by the Companies Bill in UK. Narayanan (1994) illustrates that under the proportionate liability regime, the auditor pays only his share of the damages, so his litigation cost is more sensitive to his effort; hence, he has greater incentive to minimize his litigation cost by working harder. Radhakrishnan (1999) finds that the presence of the recovery friction, a portion of any damages awards by the court retained by the lawyer as a contingent fee, leads to second-best effort by the auditor and the manager. Hillegeist (1999) demonstrates that a change in a proportional auditors’ liability can decrease audit failure rate, despite the concurrent decline in audit quality. In contrast, Patterson and Wright (2003) consider the case in which the probability of detecting a misstatement depends on the auditor’s effort and on the evidence of fraud. They investigate whether or not the prescription of large marginal liability relief for audit quality can reduce audit risk relative to joint and several liability when the audit evidence of fraud is inconclusive. Large marginal liability relief increases audit effort, reduces the auditor’s expected liability cost, and in turn, provides less incentive for the auditee to reduce the fraud rate. In the meantime, the auditor’s evidence evaluation becomes more liberal which increases the auditor’s expected liability costs and provides more incentive for the auditee to reduce the fraud rate. In their equilibrium model of fraud, the net effect of these two forces can motivate the auditee to commit fraud more often and audit risk increases.

The analysis proceeds as follows: The setting is described in Section 2.1 and Section 2.2, respectively. The existence, uniqueness and characterization of the equilibrium with auditing are established in Section 3. The empirical predictions are developed in Section 4. We conclude in Section 5. Proofs are relegated to the Appendix 1.
2 The Model

2.1 The Setting with No Auditing: The Lemons Investment Model

In this section we describe a variation of the Myers-Majluf investment “lemons” model in which a firm foregoes a positive net present value project due to asymmetric information. Then in the following section we introduce a public auditor who, through attestation of the firm’s financial report, serves to mitigate the lemons problem.

The risk neutral firm is privately informed about the value of its existing equity (defined as assets minus liabilities). The value of existing equity is denoted \( w_t \) (where subscript \( t \in \{h, l\} \) denotes high or low values and \( p(h) \) denotes the common knowledge prior probability of the value being high). We refer to the firm with high (low) value of existing equity as the high (low) type. (Since all parties are risk neutral, all results go through if the value of existing equity is stochastic with mean \( w_t \).)

The firm also has a valuable new investment project that yields positive expected net present value in excess of the initial investment, denoted \( E[x] > 0 \).\(^{12}\) Unlike Myers-Majluf, we assume that there is common uncertainty regarding the present value of the new project. The firm must immediately issue equity in order to fund the project, otherwise the opportunity is lost. (The results of this paper go through if the firm issues risky debt and is better informed about its bankruptcy risk than are investors.) The assumption that investment cannot be postponed captures the idea that a project loses value if funding is delayed. The new issue amount, denoted \( I \), represents the required investment net of any funding the firm has available in the form of cash, marketable securities, risk free debt, and solicitations from existing shareholders. We assume that existing assets are necessary for undertaking the project, so the project cannot be spun off as a separate firm. The project is further assumed to be indivisible. The firm is assumed to invest whenever it issues new equity \( I \). When we say the firm invests (does not invest), we will mean that the firm both issues and invests (neither issues nor invests).

If the firm invests in the project and the firm’s type were known to investors, the full information market value of the type \( t \) firm is \( V_t = w_t + E[x] + I \). If it does not invest, the no-investment value of firm type \( t \) is the value of its existing equity \( w_t \). If both types of firms are assumed to invest and investors do not know

\(^{12}\)Myers and Majluf (1984) effectively made the same assumption by arguing that the firm would have discarded an investment opportunity with negative present value before seeking new investment from investors (page 191).
the true value of the firm’s existing equity, investors assess the firm’s no-information market value as

\[ V_\phi = E[w|\phi] + E[x] + I, \]

where \( E[w|\phi] = p(h)w_h + (1 - p(h))w_l \) and the \( \phi \) denotes the fact that investors have no information when forming expectations. Since investors are assumed to be risk neutral in a competitive market and the risk free rate of return is assumed to be zero, investors provide \( I \) in exchange for the fraction \( I/V_\phi \) of the firm. That is, investors expect a return of \( \frac{I}{V_\phi} \cdot V_\phi = I \) from an investment of \( I \). The existing shareholders retain fraction \( (V_\phi - I)/V_\phi \) of the firm; their retained firm value as a function of the firm’s market value is denoted

\[ v_t[V_\phi] = \frac{V_\phi - I}{V_\phi} (w_t + E[x] + I). \]

As in Myers-Majluf model, we assume that the firm invests if the retained firm value exceeds the no-investment firm value. That is,

\[ v_t[V_\phi] > w_t. \]

To see the trade-off faced by the firm in deciding whether or not to invest, note that the retained firm value can be restated as follows:

**Remark 1** Retained Firm Value: The retained firm value of the type \( t \) firm is

\[ v_t[V_\phi] = w_t + I \left\{ \frac{E[x]}{I} - w_t - E[w|\phi] \right\}. \]

The first term is the no-investment value of the firm while the second term is the expected return of the new project in excess of the initial investment. The first term in the brackets is the rate of return on the new investment while the second term in the brackets is the cost of capital required by new investors when they do not know the true value of the firm’s existing equity.\(^{13}\) In other words, the second term is the lemons

\(^{13}\) We define the cost of capital as the expected return on the firm market value. The denominator of the second term in the brackets is the firm market value when there is no information available to investors. The numerator is the expected return, which is the difference between the actual firm value and the expected firm value. The definition is consistent with standard asset pricing models in finance (e.g., Fama and Miller (1972)) and disclosure models in accounting (see Lambert et. al (2007)).
premium above the zero rate of return required by investors. Note that if the firm’s type were known to investors the cost of capital is zero and all new projects would be undertaken.

In our model, a lemons problem occurs when the project is undertaken by the low type but not the high type. Note that the low type always invests because its cost of capital is negative, that is, \((w_l - E[w|\phi]) / V_\phi < 0\). To have a lemons problem, we assume that the retained firm value for the high type firm is less than the no-investment firm value:

\[
w_h > v_h[V_\phi]\quad(1)
\]

In other words, the cost of capital for the high type firm exceeds the new project’s expected rate of return. That is,

\[
\frac{w_h - E[w|\phi]}{V_\phi} > \frac{E[x]}{I}\quad(2)
\]

Since the firm with high value of existing equity does not undertake the valuable project, the expected level of new investment is \((1 - p(h))I\) and the expected lost investment is \(p(h)I\).

Examination of assumption (2) indicates that the underinvestment problem occurs ceteris paribus when the expected returns \(E[x]\) are too low, the required equity issue \(I\) is too large, the probability of the high type is too small, or the variance between high and low values, \(w_h - w_l\), is too high. Furthermore, assumption (2) indicates that the cost of capital for the high type firm is decreasing in investors’ perceptions of the firm’s existing equity value. If the high firm could improve investors’ perception of its existing equity value sufficiently, then the firm’s cost of capital could be sufficiently reduced that the new project would be undertaken.

In the absence of a mechanism to penalize misstatements, both types would report high and the report would not be credible. Consequently, investors would assess the firm as having the no-information market value and, given the assumption in (2), the underinvestment problem persists. In the next section, we examine the situations where a mandatory audit is required and how an audit may mitigate the underinvestment problem.
2.2 The Setting with Auditing

In this section, we focus on attestation by an auditor as the mechanism by which the firm can credibly communicate its value. Previous analysis focusing on auditors in financial markets have studied the information transmitted to the market by the choice to hire an auditor, the choice of auditor type, or the choice of auditor fees. In contrast, we focus on the information content of the auditor’s attestation of the firm’s financial report regarding its existing equity. In order to highlight the information content of attestation, we abstract from other informational roles served by auditors by assuming a single period setting in which auditing is mandatory, there is only one auditor type, and the audit fee is unobservable. In a single period setting, repetition, learning, and reputation of the firm and auditor are not salient. Because auditing is assumed mandatory, the firm cannot signal its type through the decision to hire an auditor (as in parts of Melumad and Thoman (1990)). Since there is only one type of auditor, no signaling is possible by the firm’s choice of auditor (as in Titman and Trueman (1986)). Furthermore, as the firm does not have spare capital unless the project is funded by investors, it cannot choose the level of investment to signal its type either.

In our model, the firm hires the auditor and pays him an audit fee $F$ that we assume is unobservable by investors.\footnote{We assume that an audit fee $F$ is unobservable by investors for ease of exposition. However, this assumption is not crucial. Though $F$ differs across the different equilibria, observing $F$ will not impact our results since the amount of $F$ is uninformative about the auditee’s type. As we illustrate later, $F$ is endogenously determined ex ante for a given equilibrium. Moreover, we note that this assumption is in line with common IPO practice. When firms go public, disclosure of IPO audit fees is only voluntary and many firms do not disclose it. Consequently, empirical studies of auditing in IPO’s (e.g., Betty (1993), Willenborg (1999), and Mayhew and Wilkens (2003)) use non-underwriting expenses or accounting fees that issuing firms pay in the IPO as a proxy for the audit fees. According to the SEC guidelines, this amount may be estimated and include all payments to the audit firms of issuing firms.} We assume a competitive market for audit clients so the auditor receives a zero profit. Thus, the audit fee $F$ just equals the auditor’s costs of sampling $C_s$ and expected costs of audit errors ($C_1$ and $C_{II}$) which will be specified in detail later.

The firm privately observes the value of its existing equity (i.e., its type), and privately submits an unaudited report of the value of its equity to the auditor for attestation. That is, the firm’s type is unobservable to the auditor and the investors, while the unaudited report is unobservable to the investors. Denote the two possible private reports as the unaudited high report $\hat{h}$ or the unaudited low report $\hat{l}$. The firm’s reporting strategy is denoted as $\rho(\hat{t}|t)$, the probability of reporting $\hat{t} \in \{\hat{h}, \hat{l}\}$ when $t \in \{h, l\}$ is the true type.

The auditor tests the hypothesis that the firm’s unaudited report $\hat{t}$ is equal to its true type $t$. He collects audit evidence from the firm’s financial records incurring cost of sampling $C_s$. The auditor’s audit evidence
is an informative signal $\omega$ whose conditional distribution $p(\omega|t)$ is correlated with the true value of existing equity and independent of the new project’s future returns.\footnote{Independence is a simplifying assumption that is not critical for our results.} The audit evidence is private information to the auditor, that is, it is unobservable to the firm and investors.

We consider the audit technology $\{p(\omega|t), t \in \{h, l\}\}$ consisting of discrete conditional distributions, which is called a discrete audit technology.\footnote{For tractability, we derive our predictions for discrete distributions. There is very little loss in generality in confining attention to discrete audit technologies because, as Shibano (1990) shows, any audit technology can be approximated arbitrarily closely in an informational sense by a discrete audit technology.} The audit technology $\{p(\omega|t), t \in \{h, l\}\}$ captures all information about existing equity that is available solely to the auditor due to his expertise in observing and interpreting financial records and valuing assets and liabilities. We assume that the audit technology is imperfect in that there is no audit evidence from which the auditor can infer the true type with certainty. Further, we assume that the audit technology is informative in the sense of strict first-order stochastic dominance (FOSD), that is, $\int_{\omega} p(\omega|t=h) d\omega < \int_{\omega} p(\omega|t=l) d\omega$.

Based on the audit evidence $\omega$, the auditor either accepts (denoted $a$), or rejects (denoted $r$), the unaudited report $\hat{t}$. The decision rule for acceptance or rejection is denoted $\delta(d|\hat{t}, \omega)$, that is, the probability of making decision $d \in \{a, r\}$ given the unaudited report is $\hat{t}$ and the audit evidence is $\omega$. Shibano (1990) shows that the ex post problem of accepting or rejecting the firm’s unaudited report after observing the audit evidence is (almost everywhere) equivalent to the ex ante problem (i.e., before observing the audit evidence) of choosing the probability $\alpha_{\hat{t}}$ of rejecting a true unaudited report, called the Type I error probability, and the probability $\beta_{\hat{t}}$ of accepting a false unaudited report, called the Type II error probability.\footnote{Patterson (1993) extends the results of Shibano (1990) to include the choice of sample size by the auditor. Alternatively, the auditor may exert costly effort to increase the precision of the audit technology. We assume that the auditor simply picks a fixed sample size that meets minimum auditing standards and consequently the only strategic decision is the choice of accept/reject regions. In other words, we focus on the auditor’s trade-off between type I and type II errors given a fixed sample size (fixed audit evidence) and a fixed level of precision of audit technology.} Furthermore, Shibano (1990) shows that $\beta_{\hat{t}}$ is uniquely determined by $\alpha_{\hat{t}}$ according to a well-behaved functional relationship $\beta_{\hat{t}} = B_{\hat{t}}[\alpha_{\hat{t}}]$.

If the auditor rejects, we assume that the firm undertakes the audit adjustments required by the auditor, revises its unaudited financial report to satisfy the auditor, and privately (i.e., unobservable to investors) incurs an audit adjustment cost $R \geq 0$.\footnote{In effect, we are assuming that the auditor retains all the bargaining power over audit adjustments (see Antle and Nalebuff (1991)). None of our results are sensitive to this assumption as long as the auditor has some bargaining power.} Audit adjustment costs include the costs of adjusting the firm’s
financial records to conform with the adjustments required by the auditor, the costs of additional evidence collection undertaken in response to evidence that is inconsistent with the firm’s unaudited report, and the costs of audit delays.

The auditor releases a public report that may be either an audited high report $\hat{H}$ or an audited low report $\hat{L}$. In practice, both the initial unaudited financial report from the firm to the auditor as well as the adjustments required by the auditor prior to the public release of the audited financial statements are confidential. To capture the confidentiality of the auditor-firm interaction, we assume that the audited high report $\hat{H}$ is released if the unaudited high report was accepted or if the unaudited low report was rejected. Similarly, the audited low report $\hat{L}$ results from the unaudited low report being accepted or from the unaudited high report being rejected.$^{19}$

Investors observe only the audited report and update their prior expectation of the firm’s market value using rational expectations. That is, investors revise beliefs using the equilibrium reporting strategy of the firm $\{\rho(\hat{h}|h), \rho(\hat{h}|l)\}$ and the errors inherent in the audit evidence and the equilibrium audit decision rule $\{\alpha_h^*, \alpha_l^*\}$ and those beliefs are fulfilled in equilibrium. Their assessment of the firm’s market value, denoted $V_T$, is conditional on the audited report $\hat{T}$, where $\hat{T} \in \{\hat{H}, \hat{L}\}$, and on the firm’s equilibrium investment decision. Investors’ assessment determines the firm’s cost of capital. Comparing the cost of capital to the project’s rate of return, the firm then decides whether to undertake the investment. Given that, we then characterize the firm’s objective function. First, based on Remark 1 and the payoff specification in Section 2.2, we can show the following

**Remark 2** The firm of type $l$ chooses a reporting strategy $\rho(\hat{h}|l)$ to maximize its objective function:

$$
\max_{\rho(\hat{h}|l)} \rho(\hat{h}|l)[(1 - B_h^*[\alpha_h^*])(v_l[V_L] - R - F) + B_h^*[\alpha_h^*](v_l[V_H] - F)]
+ (1 - \rho(\hat{h}|l))(v_l[V_L] - F),
$$

where $v_l[V_T] \equiv \frac{V_T - I}{V_T} (w_t + E[x] + I)$ is the type-$t$ firm’s retained firm value, given an audit report $\hat{T} \in \{\hat{H}, \hat{L}\}$ and before the payment of the audit fee $F$.\footnote{We could accommodate the possibility of a publicly announced reject opinion in our model by having a small probability that, when the auditor privately rejects, the negotiations over audit adjustments break down and the rejection is publicly announced. The qualitative results in this paper would not change.}

19
The first line of (3) is the expected utility to the low type firm if it reports high. With probability $(1 - B_h[\alpha_h])$, the auditor correctly rejects the high report so the low type receives $v_l[V_L]$ and the firm pays an audit adjustment cost $R$. With probability $B_h[\alpha_h]$, the auditor commits a type II error, falsely accepting the high report and so the low type receives $v_l[V_L]$ and the firm pays an audit adjustment cost $R$. Since the high type has no incentive to mimic the low type, only the low type is believed to report low and the auditor always accepts a low report (i.e., $\alpha_l = 0$ and $B_l[\alpha_l] = 1$). Thus, when the low-type firm reports a low report, it receives $v_l[V_L]$ without having to pay a rejection cost. Second, the firm of type $h$ chooses $\rho(h|h)$ to maximize:

$$Max_{\rho(h|h)} \rho(h|h)[(1 - \alpha_h)(v_h[V_H] - F) + \alpha_h(w_h - R - F)] + (1 - \rho(h|h))(w_h - F).$$

(4)

The maximization problem in (4) can be interpreted similarly. The expression in the square brackets in (4) represents the expected utility to the high firm if it reports truthfully. With probability $\alpha_h$, the high report is rejected incorrectly by the auditor, and the firm pays the audit adjustment cost $R$. With probability $(1 - \alpha_h)$, the auditor accepts the firm’s high report and the firm receives $v_h[V_H]$.

There are two types of auditor liability. First, investors in a firm with an audited high report may discover the firm’s true type was low. If those investors had paid more than the value of a low type firm, they may sue the auditor for damages. In other words, the failure by the auditor to detect a misstatement in the firm’s audited report, referred to as an audit failure, may result in a liability cost to that auditor. In this case, we assume that the auditor is assessed a penalty, $A$, to be paid to investors as a damage award. As we illustrate later on (see Remark 5), investors incorporate this damage award into their market value of the firm.

Because we are interested in studying the effect of varying the size of the damage payment on the resulting

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20 The detailed analysis is presented in the proof of Proposition 1.

21 Audit adjustment costs includes the costs of adjusting the firm’s financial records to conform with the adjustments required by the auditor, the costs of additional evidence collection undertaken in response to evidence inconsistent with the firm’s unaudited report, and the costs of audit delay. Our analysis assumes away the effect of new information acquired by the additional evidence and that audit adjustment costs are not a function of the firm’s report. Our results are not qualitatively sensitive to these simplifying assumptions.

22 We assume that investors can sue the auditor but not the firm. This assumption is adopted for two reasons. First, this assumption simplifies the analysis, permitting us to avoid questions of how damage payments are to be divided between a firm and its auditor. In addition, it makes the signaling problem more significant because the court system cannot directly punish a firm for lying. However, since the audit fee is determined competitively, any expected damage payment will be reflected in the fee; thus, ex ante the auditor’s expected utility does not depend on the way damages are assigned. Further, we assume that penalties are imposed on the auditor but not on the firm. In general, both the auditor and firm are subject to penalties if misstatements are not detected by the auditor. However, in an audit failure, the firm is often bankrupt, while the auditor has “deep pockets.” Assuming penalties on the auditor but not the firm is an extreme case capturing this differential ability to pay the penalty. This assumption is not uncommon in the literature (see, e.g., Dye (1993, 1995), Chan and Pae (1998), and Schwartz (1997), and Laux and Newman (2008)).
equilibria, we assume $A$ is exogenous; in other words, the damage award $A$ may not perfectly reflect the size of the actual damages incurred (see also Melumad and Thoman (1990)). In addition, the auditor may incur legal cost, $L$, imposed through the court, legal fees, and opportunity costs of time spent in court. This legal cost $L$ will not be recovered by investors and is considered as the cost of litigation frictions (see Laux and Newman (2009)). In this paper, we denote the expected audit liability due to an audit failure by the cost of a type II error cost, $C_{II} = A + L$.

Second, if the auditor erroneously rejects a high type who has reported high, the auditor incurs expected losses associated with the possibly being replaced as well as reputational cost. We denote the expected liability under this circumstance as, $C_I$, and refer to it as the cost of a type-I error. Our analysis focuses on policies affecting Type-II costs, and thus we take Type I costs to be exogenously given.

The auditor’s problem can be rewritten by incorporating the auditor’s liability into (3). As per Remark 2, the auditor examining the firm’s report $\hat{h}$ chooses $\alpha_{\hat{h}}$ to maximize:

$$\max_{\alpha_{\hat{h}}} \quad p(h)\rho(\hat{h}|h)[(1 - \alpha_{\hat{h}})(F - CS) + \alpha_{\hat{h}}(F - CS - C_I)]$$

$$+ (1 - p(h))\rho(\hat{h}|l)[(1 - B_{\hat{h}}[\alpha_{\hat{h}}])(F - CS) + B_{\hat{h}}[\alpha_{\hat{h}}](F - CS - C_{II})]$$

In expression (5), the auditor receives his fee net of sampling costs, $F - Cs$, in all cases. But if he rejects (accepts) the high report from the high (low) type, he pays an additional Type I error cost $C_I$ (Type II error cost $C_{II}$). Again, since only the low type is assumed to report low, the auditor always accepts a low report (that is, $\alpha_{\hat{l}} = 0$, and $B_{\hat{l}}[\alpha_{\hat{l}}] = 1$).

The equilibrium is specified by the reporting and investment strategies for the firm, $\{\rho(\hat{h}|t), \{\text{invest, not}}$
invest}, the audit opinion strategy for the auditor, \{\delta(\hat{d}, \omega)\}, and the equilibrium market valuation by investors \{V_T, \hat{T} \in \{\hat{H}, \hat{L}\}\}. See Figure 1 for a timeline of the setting with auditing and Appendix 2 for a summary of the notation.

<table>
<thead>
<tr>
<th>Audit Engagement Stage</th>
<th>Public Offering Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm hires auditor type {h, l}; reports \hat{H} or \hat{L} to auditor</td>
<td>Investors assess firm value \hat{V}_T and the firm decides whether or not to invest</td>
</tr>
<tr>
<td>Firm observes \omega from p(\omega</td>
<td>d); accepts or rejects the firm’s report;</td>
</tr>
<tr>
<td></td>
<td>Firm type revealed; aud. loses CII if false accept and loses CII if false reject</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 1: Timeline of events

3 Equilibrium Analysis

3.1 Characterization of the Equilibrium

The existence of equilibria involves a complex relationship between the behavior of the auditor and the firm during the audit engagement and investors’ expectations regarding firm value based on the information content of the audited report. The reporting strategy of the firm both affects and depends on investors’ expectations. The auditor’s decision rule affects investors’ expectations and simultaneously depends on, and affects, the firm’s behavior.

The following proposition 1 demonstrates that the equilibrium in our model always exists, is unique, and can be characterized in closed form. The equilibrium takes one and only one of three forms. In the ineffective-auditing equilibrium, the auditor is ineffective at mitigating the underinvestment problem. The detection equilibrium involves the auditor effectively reducing the underinvestment problem while detecting but not deterring overstatements by the low type firm. Finally, in the deterrence equilibrium, the auditor decreases underinvestment while detecting and deterring overstatements. Since the auditor is effective in
reducing the underinvestment problem in both the detection and deterrence equilibria, we refer to settings in which one of these equilibria hold as effective-auditing settings.

**Proposition 1** *Existence, Uniqueness, and Characterization of Equilibria*: The equilibrium in the lemons setting with auditing always exists, is unique, and is either an ineffective-auditing equilibrium, a detection equilibrium, or a deterrence equilibrium.

**Proof.** See Appendix 1 for all proofs. ■

In the remainder of this section we describe the behavior predicted in each form of equilibrium. More precise statements and mathematical expressions are found in Appendix 1. The equilibrium strategies of the auditor and the firm for each type of equilibrium are summarized in Table 1.

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Ineffective auditing equilibrium</th>
<th>Detection equilibrium</th>
<th>Deterrence equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>High type firm’s unaudited report</td>
<td>$h$</td>
<td>$h$</td>
<td>$h$</td>
</tr>
<tr>
<td>Low type firm’s unaudited report</td>
<td>$h$</td>
<td>$h$</td>
<td>$h$ with prob. $p(h</td>
</tr>
<tr>
<td>Auditor’s decision rule given unaudited high report</td>
<td>$\alpha_h^*$ (See equation (A-5))</td>
<td>$\alpha_h^*$ (See equation (A-5))</td>
<td>$\alpha_h^{**}$ (See equation (A-9))</td>
</tr>
<tr>
<td>Auditor’s decision rule given unaudited low report</td>
<td>$\alpha_l = 0$. Belief: $prob(l</td>
<td>l) = 1$</td>
<td>$\alpha_l = 0$. Belief: $prob(l</td>
</tr>
<tr>
<td>Investment decision of high firm w/ audited high report</td>
<td>Do not invest</td>
<td>Invest</td>
<td>Invest</td>
</tr>
<tr>
<td>Investment decision of high firm w/ audited low report</td>
<td>Do not invest</td>
<td>Do not invest</td>
<td>Do not invest</td>
</tr>
<tr>
<td>Investment decision of low firm w/ audited high report</td>
<td>Invest</td>
<td>Invest</td>
<td>Invest</td>
</tr>
<tr>
<td>Investment decision of low firm w/ audited low report</td>
<td>Invest</td>
<td>Invest</td>
<td>Invest</td>
</tr>
</tbody>
</table>

Table 1: Equilibrium Strategies
3.1.1 Ineffective-auditing Equilibrium

This form of equilibrium is named “ineffective-auditing” because the auditor is ineffective at reducing the underinvestment problem; only the low type invests and investment by the high type is completely lost.

In the ineffective-auditing equilibrium, both firm types provide unaudited high reports to the auditor. In response to the unaudited high report, the auditor may accept or reject the report, depending on the evidence observed. The equilibrium rate of rejection $\alpha^*_h$ is specified in (A-5) in the Appendix and is strictly less than one. On the other hand, in response to the unaudited low report, the auditor believes the firm is of low type and always accepts that report, regardless of the audit evidence. (The assumed beliefs regarding an out-of-equilibrium unaudited low report are robust in that they survive all equilibrium refinements.)

In response to the audited high report, investors (if they expect that both the high and low types with the audited high report invest) revise their valuation of the firm upward and therefore revise the firm’s cost of capital downward relative to the no information values in Section 2.1. In the ineffective-auditing equilibrium, the revised cost of capital (assuming both types invest) is still too high to induce the high type to invest. Therefore, rational investors know that, if a firm with the audited high report invests, it must be a low type. In equilibrium, the cost of capital for a firm with the audited high report that invests is based on the expectation that the firm has the low value equity. Thus, the high type does not invest and the low type does.

Investors’ response to the audited low report is similar. The revised cost of capital (assuming both types invest) is too high to induce the high type to invest. The only type to invest with the audited low report is the low type. Therefore, similar to the lemons setting without auditing, only the low type invests and thus the auditor is ineffective at mitigating the underinvestment problem.

3.1.2 Detection Equilibrium

The detection equilibrium has three similarities to the ineffective-auditing equilibrium. First both firm types submit unaudited high reports and the auditor rejects the unaudited high report with equilibrium probability $\alpha^*_h$ specified in (A-5). Second, if the auditor receives the out-of-equilibrium unaudited low report, he believes the firm is low and always accepts. (These beliefs are robust to standard equilibrium refinements.)
Finally, the low type invests in response to both the audited high report and the audited low report.

While investors revise their valuation of the firm upward and, consequently, the firm’s cost of capital downward under both the ineffective-auditing equilibrium and the detection equilibrium, the revised cost of capital in the latter case is low enough to induce the high type with the audited high report to invest. Therefore, under a detection equilibrium, investors can rationally expect that the firm with the audited high report is a high type with positive probability consistent with the auditor’s equilibrium decision rule (whereas under the ineffective-auditing equilibrium, investors know that if a firm with an audited high report invests, it must be a low type). In equilibrium, the high type with the audited high report invests; the auditor’s report is effective in partially reducing the underinvestment problem. The name “detection equilibrium” is given to this equilibrium because the auditor is effective in partially “detecting” the low type who overstates but the auditor is ineffective at “deterring” the low type from overstating.

3.1.3 Deterrence Equilibrium

The deterrence equilibrium differs from the detection equilibrium in two ways. First, the low type does not always overstate. The low type is “deterred” from always overstating by the expected costs of having the unaudited high report rejected. This deterrence effect of the auditor involves a complex equilibrium interaction between the firm’s reporting strategy, the auditor’s decision rule, and the market’s valuation process. Second, the auditor accepts or rejects the unaudited high report with equilibrium probability \( \alpha_h^* \) specified in equation (A-9) in the Appendix.

Similar to the detection equilibrium, the deterrence equilibrium involves the high type always reporting high, the auditor always accepting the unaudited low report, the low type always invests, and the high type invests only if it receives the audited high report. Therefore, the auditor is effective in partially mitigating the underinvestment problem.
4 Predicted Effect of Increases in Auditor Liability

4.1 Equilibria in Which Litigation Occurs

The motivation of this study is to shed light on the equilibrium effects of increasing auditor liability. The first step then is to focus attention only on those equilibria in which audit litigation occurs. The following remark accomplishes this step.

Remark 3 **Equilibria in Which Litigation Occurs**: Litigation occurs in the effective-auditing settings (that is, the detection and the deterrence equilibrium) but not in the ineffective-auditing equilibrium.

In general, audit legal liability depends on three key factors including the probability of misstatements in the firm’s report, the probability that the audit would fail to detect the misstatement, and the probability that the auditor would incur a legal liability due to an audit failure (Choi et al. 2008). In the ineffective-auditing equilibrium the only type of firm that invests is the low type firm. In equilibrium, investors know that if a firm invests then it must have the low value of existing equity regardless of whether the audited report is high or low. Therefore, investors require $\frac{I}{V_l}$ of the firm in exchange for investing $I$ and, after the project returns are realized, they receive $\frac{I}{V_l} \cdot V_l = I$. Since investors receive the required zero rate of return, there are no actual damages. Without actual damages, the investor lacks grounds for litigation related to an audit failure and thus, litigation against the auditor does not occur in the ineffective-auditing equilibrium.

In contrast, in the effective-auditing settings, investors value the firm with the audited high report sufficiently high to induce the high type with the audited high report to invest. After equity is issued and the investment undertaken, there is a positive probability that the firm with the audited high report is discovered to actually be a low type. Investors receive $\frac{I}{V_H} V_H$ with probability $p(h)(1 - \alpha_{\hat{h}})$ if the true type is high and $\frac{I}{V_H} V_l$ with probability $(1 - p(h)) \rho(h|l) B_{\hat{h}} \{\alpha_{\hat{h}}\}$ (where $\rho(h|l)$ may equal one) if the true type is low. In the latter case, the investors suffer a capital loss of $I - \frac{I}{V_H} V_l$. Since actual damages were sustained, the investors have grounds for litigation.
4.2 Increases in Auditor Liability in the Effective Audit Settings

Since audit liability occurs only in the effective-auditing equilibria, we focus on those equilibria in this section. The following three propositions involve predictions of increase in auditor liability in the effective-auditing settings.

4.2.1 Prediction Regarding New Investment

The key contribution of this paper is the analysis of how and why increasing auditor liability may decrease new investment. As we discussed in the introduction, what concerns many market participants is that raising legal liability for auditors may have a negative impact on the capital market. It is not surprising that the four biggest audit firms have been attempting to reduce the threat of litigation. However, the arguments typically made by market participants are qualitative and speculative in nature. In fact, the extant theoretical literature (Schwartz (1997), Newman et al. (2005)) suggests that an increase in auditors’ liability after SOX may result in increased investment. Our analysis establishes an equilibrium explanation for when and why increasing the auditors liability may decrease investment; it provides theoretical arguments consistent with the concerns described in the introduction. Specifically, Proposition 2 below establishes that when auditor’s liability for audit failure increases, the audited firms are less likely to undertake new investment.

In this model, the probability that the firm undertakes new investment is the sum of the probabilities that the high firm and the low firm invest conditional on the probability of the firm’s type. Since investors always set the cost of capital low enough to induce the low type (with either audited report) to invest, the joint probability that the firm is low and undertakes investment is simply \(1 - p(h)\). Since the high type firm only invests if it receives the audited high report, the probability of the high type investing \(p(h)(1 - \alpha_{h})\) is the joint probability of the type being high and the auditor attesting to the audited high report when the firm type is high. Note that \(\alpha_{h}\) is equal to \(\alpha_{h}^{*}\) (defined in (A-5)) in the detection equilibrium and \(\alpha_{h}^{**}\) (defined in (A-9)) in the deterrence equilibrium. The following Proposition captures the effect of increasing liability on the probability of the firm undertaking new investment.

**Proposition 2.** Effect of Increased Liability on New Investment: As auditor liability for audit failures increases, new investment decreases.

20
The intuition underlying Proposition 2 is clarified by understanding the trade-offs faced by the auditor. As discussed in Section 2.2, the auditor trades off expected losses from attesting to an understated versus an overstated report. Thus, increases in the cost of an overstatement error leads directly to the auditor rejecting the unaudited high report more often in order to reduce the likelihood of incurring the overstatement penalty. However, there is a countervailing indirect effect. In the proof of Proposition 2, we established that the more likely the auditor is to reject an unaudited high report, the more likely it is that a firm with an audited high report is indeed a high type. As investors’ valuation of the firm with the audited high report goes up, the likelihood of the low type to overstate rises, increasing the likelihood of the auditor incurring overstatement losses. Thus, it is not immediately clear that the auditor will reject a high report more often as the liability increases.

The key step in proving Proposition 2 is showing that in the effective-auditing settings, the auditor rejects more often as liability increases. That is, the direct effect always dominates the indirect effect. As the auditor increases the probability of rejecting an unaudited high report, the probability of new investment declines. To see why, note that both the low and the high types are having their unaudited high report rejected with a higher probability. Every time a high firm has its unaudited high report rejected, it receives an audited low report and finds that its cost of capital is too high.

Looking beyond this stylized setting, our analysis suggests that, as auditor’s liability increases, any firm with private information about its existing equity will find that its financial statement would more likely be understated. As long as investors cannot infer the true value from the understated reports, the firm’s valuation will be depressed, thereby increasing its cost of capital and making it less likely to pursue new, economically desirable, investment.

### 4.2.2 Prediction Regarding Audit Failure Rate

The audit failure rate is the percentage of audits involved in litigation associated with the failure to detect a materially misstated financial report which caused investors actual damages. In this model, the audit failure rate is the equilibrium joint probability, in effective-auditing settings, of the firm type being low, the low firm submitting an unaudited high report, and the auditor observing evidence that leads the auditor to erroneously

27This line of reasoning is in line with that in Gigler et al. (2009).
accept the unaudited high report.

In the detection equilibrium, the audit failure rate is 

\[(1 - p(h)) \rho(h|l) B_h [\alpha_h^*] \] 

where \(1 - p(h)\) is the prior probability of the firm being low and \(B_h [\alpha_h^*]\) is the equilibrium probability of accepting the unaudited high report from a low firm. Note that in the detection equilibrium, the probability of the low firm submitting an overstated report is one. In contrast, the deterrence equilibrium involves the low type not always overstating; the audit failure rate is 

\[(1 - p(h)) \rho(h|l) B_h [\alpha_{h}^{**}] \] 

The probability \(\rho(h|l)\) of the low type submitting the unaudited high report is an equilibrium response to the auditor’s equilibrium acceptance probability \(B_h [\alpha_{h}^{**}]\).

The following proposition answers the question of how increasing auditor liability affects the audit failure rate.

**Proposition 3** Effect of Increased Liability on Audit Failures: As auditor liability for audit failures increases, the audit failure rate decreases.

The intuition for Proposition 3 is as follows: As auditor liability for audit failures increases, overstatement errors become more costly to the auditor. In equilibrium, the auditor reacts by rejecting the unaudited high report more often. The audit technology is informative so the auditor is incrementally more likely to reject a low type than a high type. There is a decrease in the equilibrium probability that the firm with audited high report is actually a low type. Thus, in the detection equilibrium, the auditor’s increasing “conservativeness” leads directly to a lower probability of audit failure litigation.

However, in the deterrence equilibrium, there are two countervailing forces that must be considered. First, since the auditor rejects unaudited high reports more often, the firm with the audited high report is less likely to be a low type. Investors can make stronger equilibrium inferences regarding the value of the firm with the high report, and the value assigned to the firm with the audited high report is increased. An increase in the valuation of the firm with the audited high report provides more market incentive for the low type firm to report high. On the other hand, the higher rate of rejection of high reports deters the low firm from overstating. It is possible that the increase in market price might overcome the deterrence effects of the rejection costs, leading to an increased likelihood of the firm overstating. If the marginal increase in overstatement is greater than the marginal decrease in auditor error, then the rate of audit failures might
increase with auditor liability. However, it turns out in this model that the deterrence effects always outweigh the market effects.

Propositions 2 and 3 therefore suggest the following remark:

**Remark 4 Tradeoff between New Investment and Audit Liability**: As auditor liability increases (decreases), both new investment and audit failures decrease (increase).

### 4.2.3 Prediction Regarding Costs of Capital

As discussed in Section 2.1, the firm’s cost of capital is a function of the investors’ valuation of the firm. In this section we first discuss how investors value the firm and set its cost of capital and then we develop predictions regarding how the cost of capital is affected by changes in auditor liability.

In making their valuation decisions, investors have rational expectations in that they take into account both the equilibrium reporting and issuing/investing behavior of the firm and the equilibrium audit opinion. The following Remark 5 captures how investors incorporate their expectations regarding the equilibrium interaction between the auditor and firm into their valuation of the firm.

**Remark 5 Equilibrium Market Valuation**: a. In both the detection and the deterrence equilibria, the market value of the firm with the audited low report is

\[ V^L_L = V_I \equiv w_l + E[x] + I \]

b. In the detection equilibrium, the market value of the firm with the audited high report is:

\[ V^H_H[\alpha^*_h] = \gamma[\alpha^*_h] w_h + (1 - \gamma[\alpha^*_h])(w_l + A) + E[x] + I, \]

where

\[ \gamma[\alpha^*_h] = \frac{p(h)(1 - \alpha^*_h)}{p(h)(1 - \alpha^*_h) + (1 - p(h))(B_h[\alpha^*_h])}. \]
c. In the deterrence equilibrium, the market value of the firm with the audited high report is:

\[ V_{\bar{H}}^{**}[\alpha^*_h, \rho(\hat{h}|l)] = \gamma[\alpha^*_h, \rho(\hat{h}|l)]w_h + (1 - \gamma[\alpha^*_h, \rho(\hat{h}|l)])(w_l + A) + E[x] + I, \]

where

\[ \gamma[\alpha^*_h, \rho(\hat{h}|l)] = \frac{p(h)(1 - \alpha^*_h)}{p(h)(1 - \alpha^*_h) + (1 - p(h))\rho(\hat{h}|l)(B_h[\alpha^*_h])}. \]

The intuition behind Remark 5a is that investors know that in equilibrium any rational valuation of the firm with the audited low report would not be high enough to induce the high type to invest. Therefore, any firm with the audited low report that invests in equilibrium must be the low type.

In contrast, in both the detection and the deterrence equilibria, investors’ valuation of the firm with the audited high report is high enough to induce the high type with the audited high report to invest. Therefore, it is rational for investors to assign an expected value to the firm with the audited high report that is consistent with the firm’s equilibrium reporting/investing behavior and the auditor’s equilibrium decision rule.

Therefore, in Remark 5b, the market price \( V_{\bar{H}}^{**}[\alpha^*_h] \) in the detection equilibrium of the firm with the audited high report is a Bayesian weighted average of the full information high and low values, where the weight \( \gamma[\alpha^*_h] \) on the high type (weight \( 1 - \gamma[\alpha^*_h] \) on the low type) is the conditional probability, given the audited high report, of the type being high (low), the high (low) type reporting high, and the auditor not rejecting the high report from the high (low) type.

Similar to the detection equilibrium, in Remark 5c, the market price \( V_{\bar{H}}^{**}[\alpha^*_h, \rho(\hat{h}|l)] \) associated with the audited high report in the deterrence equilibrium is a Bayesian revision based on the information in the audited report. However, the weights incorporate the reduced probability that the low type overstates in equilibrium and the increased probability that the auditor rejects the high report in equilibrium.

Now we can identify the cost of capital for the firm with the audited report. Recall that in equation (2), the high firm’s cost of capital in the no-auditing world is \( \frac{w_t - E[w|\phi]}{\phi} \). The lack of information about existing equity is denoted \( \phi \) signifying the empty set. The analogous expression in the effective-auditing settings is \( (w_t - E[w|\hat{T}])/V_{\hat{r}} \), where the audited report \( \hat{T} \) could be high \( \hat{H} \) or low \( \hat{L} \) and where in equilibrium: \( E[w|\hat{L}] = w_l \) and \( E[w|\hat{H}] = \gamma[\alpha^*_h]w_h + (1 - \gamma)(w_l + A) \), where \( \gamma \) is either \( \gamma[\alpha^*_h] \) or \( \gamma[\alpha^*_h, \rho(\hat{h}|l)] \) defined in
Remark 5b and 5c, respectively. In Proposition 4, predictions regarding the cost of capital are summarized:

**Proposition 4 Effect of Increased Liability on Cost of Capital:** Increasing auditor liability for audit failures

1. does not affect the cost of capital for a firm with an audited low report.

2. decreases the cost of capital faced for a firm with an audited high report.

The effect of auditor liability changes for the audited low report is straightforward. Since the only type that invests with the audited low report is the low type, the cost of capital to the firm with that report is \( w_t - w_l \), with \( t = h, l \). This cost is invariant to the auditor decision rule and thus also invariant to any changes in the auditor’s liability.

In contrast, the cost of capital faced by the firm with the audited high report does depend on the auditor’s decision rule in equilibrium. In effective-auditing settings, the expression for cost of capital is

\[
\frac{w_t - E[w|\hat{H}]}{V_{ft}} = \frac{w_t - \gamma w_h - (1 - \gamma)(w_l + A)}{\gamma w_h + (1 - \gamma)(w_l + A) + E[x] + I},
\]

for \( t = h, l \) where \( \gamma \) equals either \( \gamma[\alpha^*_h] \) or \( \gamma[\alpha^*_h, \rho(h|l)] \) as defined in Remark 5b and 5c. The impact of higher audit liability on the high type firm’s cost of capital is ambiguous. On the one hand, it makes the auditor more conservative and more likely to reject a high report. This subsequently increases the high-type firm’s cost of capital. On the other hand, the market assigns a higher value to an audited high report because investors know that the report is more likely to have been issued by the high-type firm, thereby reducing its cost of capital. Thus, a change in the auditor liability influences both the equilibrium audit decision rule and the firm’s reporting, and subsequently affects the cost of capital.

The surprising feature of Proposition 4-2 is that by construction the expected returns to investing in the firm with the high report is constant and equal to the investors’ required rate of return. However, each type of firm that has the audited high report faces a lower cost of capital as auditor liability increases. How can this happen?

This paradox is variation of Simpson’s Reversal Paradox studied in Sunder (1983). In Simpson’s Paradox, a weighted average is made of weighting on two factors. The paradox occurs when the factors decrease
but their weighted average increases. This seemingly paradoxical combination of effects is, of course, explained by the fact that weighting on the factors change is a precise way. The increased weighted average results from putting sufficiently higher weight on the larger of two (now reduced) factors.

In our model, the expected return to investors from investing in the firm with the audited high report is constant because investors are assumed competitive and are assumed to require a zero rate of return. At the same time, the cost of capital faced by each type of firm with the audited high report is decreasing. This is explained by the fact that the weighting on the high type firm with the high report is increased.

The distinctive feature of the Simpson Paradox in our model is that it is endogenously generated by the auditor’s optimal use of audit evidence in response to increase in auditor liability. As liability increases, the auditor rejects the high report with greater frequency but he does not do this “blindly” with respect to the audit evidence. If an auditor simply rejects a high report more often unconditional on the audit evidence, the conditional probability of the firm being high given that it has received the audited high report stays constant. Therefore, the cost of capital for the firm with the high report would stay constant.

Instead, the auditor uses the information in the audit evidence to reject the high report more often when the audit evidence indicates that the unaudited high report is most likely to be coming from the low type. Consequently, the incremental probability of rejecting a high report from the low type is higher than the incremental probability of rejecting a high report from the high type. As a result, the conditional probability of the firm type being high given the audited high report increases. In this model, Simpson’s Paradox arises as an endogenous response to higher liability and the optimal use of audit evidence. The apparent paradox is resolved by the observation that increasing auditor liability leads the auditor to reject both types more often, but he optimally uses his information to reject the high type incrementally less often than the low type. Thus the expected return to investing in the firm with the audited high report is constant but the cost of capital for each firm type with the audited high report decreases. We highlight this result in Corollary 1:

**Corollary 1 Effect of Increased Liability on Audit Conservatism:** As auditor liability for audit failures increases, the auditor becomes more conservative.

Proposition 4-2, together with Corollary 1, confirms the intuition that, in response to increased auditor liability, new investment falls not due to an increase in the cost of capital. Instead, investment declines
because the auditor becomes more conservative and therefore more likely to certify understated audited reports for the high type firm.

5 Concluding Comments

Capital market participants and regulators may be interested in the economic trade-offs documented in our model. On the one hand, imposing higher legal liability may prevent the occurrence of audit failures, a desired consequence for the Sarbanes-Oxley Act of 2002. On the other hand, as legal liability increases, new investment may be reduced because of the information asymmetry between the firm and new investors, an undesirable outcome from an efficiency viewpoint. Our study identifies and characterizes this economic trade-off, but is silent on how to resolve the tradeoff. Future research may consider positing a social welfare function that trades off the disutility of audit failures versus the utility of new investment and thereby determining the socially optimal level of liability.

Our study can be extended in several ways. First, while this paper intends to address an unintended consequence of high audit liability, SOX impact is much broader. For example, Section 201 of SOX establishes auditor independence standards, and limits the extent of non-audit services. This limitation could be a mixed blessing from investors’ perspective. Non-audit and audit services may be synergetic, and thus improve profitability and increase the value of audit attestation. On the other hand, they may also reduce auditor independence and possibly adversely affect the value of attestation. Consequently, it is not immediately clear how imposing higher audit liability would affect investors’ incentive to invest in capital markets (see Lu and Sapra (2009)).

A second extension would be to allow for heterogeneous audit quality. One may measure audit quality by the extent to which the market value of a firm is increased by an audit report (see Dye (1993)). One the one hand, the post-audit market value increases in the probability of rejecting an unaudited high report, which consequently discourages the low type from overstating its report. On the other hand, higher rejection probability leads to higher market value for a firm with an audited high report, and thus a low type would possibly be more willing to submit an unaudited high report. Thus, the net effect of higher audit quality, measured by the probability of rejecting an unaudited high report, is not immediately clear as it is a function of the relative magnitude of two opposite effects.
A third extension would be to allow the auditor to choose the sample size (as in Newman and Noel (1989), Shibano (1990), and Patterson (1991)). In this case, higher auditor liability would induce two countervailing effects. First, the auditor would choose a more precise, and yet more costly, audit technology (i.e., larger sample size) which might lower both Type I and Type II error rates. Second, for the chosen audit technology, the auditor would trade off Type I and Type II errors in response to the new relative costs of the errors. The net effect would depend on the specific assumptions about the audit technology and cost function of the sample size. The countervailing forces may make predictions less sharp, but we conjecture that the qualitative results of this paper do not disappear in the expanded model.

Finally, our analysis can be extended to allow for a positive correlation between the investment return and the existing equity value. For example, one may assume that the expected present value of the investment could be negative for the low-type firm \( E(x_l) < 0 \) and positive for the high-type firm \( E(x_h) > 0 \). This assumption would exacerbate the lemons problem, because the dilution cost for the high type would be higher compared with that under the current setting. If the cross-subsidization resulting from the information asymmetry is sufficiently larger than the magnitude of the (negative) benefit of the low type firm’s investment, the latter might choose to invest in the negative-NPV project leading to avoidance of profitable investments by the high type firm. We conjecture that the three equilibria aforementioned would still exist but ineffective auditing equilibrium would be more likely to occur.

These potential extensions are left for further research.
Appendix 1: Proofs

The results that follow depend on the general characterization of the relationship between error probabilities presented in Proposition 0 below:

**Proposition 0** (Shibano (1990)): The choice among decision rules $\delta(d|\hat{t}, \omega)$ is payoff-equivalent to the choice among error probabilities $(\alpha_{\hat{t}}, \beta_{\hat{t}})$, where $\hat{t}$ is the null hypothesis.

**Proof.** A nondegenerate audit technology is defined as a pair of distributions $\{p(\omega|t), t \in \{h, l\}\}$ such that $p(\omega|h) \neq p(\omega|l)$ on a set of $\omega$ of positive measure. An audit technology $\{p(\omega|t), t \in \{h, l\}\}$ has constant support if the set of $\omega$ that have positive probability when the type is $h$ is identical to the set of $h$ that have positive probability when the type is $l$. That is, $\{z|p(\omega|h) > 0\} = \{z|p(\omega|l) > 0\}$.

For any nondegenerate audit technology with a constant support,

(i) The auditor can confine his choice of tests to the class of tests of the null $\hat{t}$ versus the alternative $\hat{t}'$ represented as an error function $B_{\hat{t}}[\alpha_{\hat{t}}]$ mapping $[0,1]$ to $[0,1]$ where $B_{\hat{t}}[\alpha_{\hat{t}}]$ is convex and strictly decreasing on $[0,1]$ and differentiable for almost every $\alpha_{\hat{t}} \in [0,1]$. $B_{\hat{t}}[\alpha_{\hat{t}}]$ is the test with the lowest Type II error probability for tests with Type I error probability $\alpha_{\hat{t}}$.

(ii) There exists a well-defined inverse function $B_{\hat{t}}^{-1}[\alpha_{\hat{t}}]$ mapping $[0,1]$ to $[0,1]$. $B_{\hat{t}}^{-1}[\beta_{\hat{t}}]$ is convex and strictly decreasing on $[0,1]$ and differentiable for almost every $\beta_{\hat{t}} \in [0,1]$. $B_{\hat{t}}^{-1}[\beta_{\hat{t}}]$ is the lowest Type I error probability for tests with Type II error probability $\beta_{\hat{t}}$.

(iii) $B_{\hat{t}}[0] = 1$ and $B_{\hat{t}}[1] = 0$. Furthermore, $B_{\hat{t}}^{-1}[0] = 1$ and $B_{\hat{t}}^{-1}[1] = 0$.

(iv) $-B_{\hat{t}}'[\alpha_{\hat{t}1}] \geq -B_{\hat{t}}'[\alpha_{\hat{t}2}]$ if and only if $\alpha_{\hat{t}1} \leq \alpha_{\hat{t}2}$.

(v) $1 - \alpha_{\hat{t}} - B_{\hat{t}}[\alpha_{\hat{t}}] > 0$, $\forall \alpha_{\hat{t}} \in (0, 1)$. When this condition holds, we say that the audit technology discriminate between types.

(vi) $B_{\hat{t}}'[\alpha_{\hat{t}}](1 - \alpha_{\hat{t}}) + B_{\hat{t}}[\alpha_{\hat{t}}] < 0$, $\forall \alpha_{\hat{t}} \in (0, 1)$.

(vii) Any function $g(\cdot)$ that maps $[0,1]$ into $[0,1]$, is everywhere convex and strictly decreasing, and almost everywhere differentiable, is an “admissible” audit technology. That is, there exists a pair of distributions $\{p(\omega|t), p(\omega|t')\}$ such that $g(\cdot)$ represents the class of tests of the null $t$ versus the alternative $t'$ under those distributions.
An audit technology \{\bar{p}(\omega|t), \bar{p}(\omega|t')\} is a discrete audit technology if both \(\bar{p}(\omega|t)\) and \(\bar{p}(\omega|t')\) are discrete probability distributions. For any arbitrary audit technology \{\bar{p}(\omega|t), \bar{p}(\omega|t')\} with error function \(B_t[\cdot]\), there exists a discrete audit technology \{\bar{p}(\omega|t), \bar{p}(\omega|t')\} with error function \(\bar{B}_t[\cdot]\) such that \(B_t[\cdot] = 0\) almost everywhere, and \(\bar{B}_t''[\cdot] = 0\) almost everywhere. Q.E.D.

Proof of Remark 1:

\[
v_t[V_\phi] = \frac{V_\phi - I}{V_\phi} (w_t + E[x] + I) = w_t + E[x] + \frac{V_\phi - I}{V_\phi} I - \frac{I}{V_\phi} (w_t + E[x]) = w_t + I \left\{ \frac{E[x]}{I} - \frac{w_t - E[w|\phi]}{V_\phi} \right\}.
\]

Q.E.D.

Proof of Remark 2:

Step 1: Consider the case where the auditor examines the unaudited high report. Let \(U(d|t, \hat{h})\) denote the auditor’s utility from decision \(d\) given true type \(t\) and unaudited report \(\hat{h}\). The ex post problem (that is, after the auditor observes \(\hat{h}\) facing the auditor examining report \(\hat{h}\), for each \(\omega \in \Omega\), is:

\[
\max_{\{\delta(d|\hat{h}, \omega), d=a,r\}} \sum_{t=1}^{T} \sum_{d=a,r} \frac{p(t) \rho(\hat{h}|t)p(\omega|t)}{p(\hat{h}|\omega)} \delta(d|\hat{h}, \omega) U(d|t, \hat{h}).
\]

Step 2: Factoring out \(1/p(\hat{h}|\omega)\) because it does not affect the maximization, the auditor’s problem becomes

\[
\max_{\{\delta(d|\hat{h}, \omega), d=a,r\}} \sum_{t=1}^{T} \sum_{d=a,r} p(t) \rho(\hat{h}|t)p(\omega|t) \delta(d|\hat{h}, \omega) U(d|t, \hat{h}).
\]

Step 3. The ex post problem in step 2 can be restated as the ex ante problem (that is, at the point in time
before he observes $\omega$) below:

$$\max_{\delta(d|h, \omega), d=a,r} \int \sum_{t=1} \sum_{d=a,r} p(t)\rho(\hat{h}|t)p(\omega|t)\delta(d|\hat{h}, \omega)U(d|t, \hat{h})d\omega.$$  

The solution of the ex ante problem is (almost everywhere) identical to that of the ex post problem because the ex ante problem involves point-wise maximization for each $\omega$.

**Step 4:** By algebraic rearrangement, an equivalent statement is

$$\max_{\delta(d|h, \omega), d=a,r} p(h)\rho(\hat{h}|h) \left\{ U(a|h, \hat{h}) \int \sum_{t=1} \sum_{d=a,r} p(t)\rho(\omega|t)\delta(a|\hat{h}, \omega)\partial\omega + U(r|h, \hat{h}) \int \sum_{t=1} \sum_{d=a,r} p(t)\rho(\omega|t)\delta(r|\hat{h}, \omega)d\omega \right\}$$

$$+ (1 - p(h))\rho(\hat{h}|l) \left\{ U(r|l, \hat{h}) \int \sum_{t=1} \sum_{d=a,r} p(t)\rho(\omega|t)\delta(r|\hat{h}, \omega)\partial\omega + U(a|l, \hat{h}) \int \sum_{t=1} \sum_{d=a,r} p(t)\rho(\omega|t)\delta(a|\hat{h}, \omega)d\omega \right\}.$$  

**Step 5:** By definition of error probability $\{\alpha_{\hat{h}}, \beta_{\hat{h}}\}$, the auditor’s maximization reduces

$$\max_{\{\alpha_{\hat{h}}, \beta_{\hat{h}}\}} p(h)\rho(\hat{h}|h) \left[ U(a|h, \hat{h})[1 - \alpha_{\hat{h}}] + U(r|h, \hat{h})\alpha_{\hat{h}} \right]$$

$$+ (1 - p(h))\rho(\hat{h}|l) \left[ U(r|h, \hat{h})[1 - \beta_{\hat{h}}] + U(a|h, \hat{h})\beta_{\hat{h}} \right].$$  

**Step 6:** By Proposition 0(i), the auditor’s problem can be equivalently stated as a choice of the optimal $\alpha_{\hat{h}}$ only.

$$\max_{\alpha_{\hat{h}}} p(h)\rho(\hat{h}|h) \left[ U(a|h, \hat{h})[1 - \alpha_{\hat{h}}] + U(r|h, \hat{h})\alpha_{\hat{h}} \right]$$

$$+ (1 - p(h))\rho(\hat{h}|l) \left[ U(r|h, \hat{h})[1 - B_\hat{h}[\alpha_{\hat{h}}]] + U(a|h, \hat{h})B_\hat{h}[\alpha_{\hat{h}}] \right].$$

The simplification of decision rule given the unaudited low report is similar. Q.E.D.

**Proof of Proposition 1:** The analysis in this section proceeds with the following 4 steps:

Step 1: The analysis proceeds using backward induction: Assuming that a deterrence or a detection equilibrium exists, we then state the player’s objective functions.

Step 2: Then we show that the only reporting strategies that appear in equilibrium of the audit engage-
ment involve both types reporting high or the high type reporting high and the low type randomizing between high and low reports. See Lemmata 1-3.

Step 3: In Lemma 4 we identify auditor decision rules that satisfy one or more conditions for existence of equilibrium and we show that each decision rule exists for any given parameters. Then in Observation 6 we state the necessary and sufficient conditions for equilibrium in terms of these auditor decision rules. In Lemma 5, we show that equilibrium exists, is unique, and is either the ineffective-auditing, detection or deterrence equilibrium.

Step 4. We provide a numerical example of existence of each of the three equilibrium forms to show that it is feasible to satisfy the conditions we identify in Lemma 5.

**Step 1:** The player’s objective functions. Recall that in the deterrence equilibrium the high type rm reports high, the low type randomizes between the high and low reports, and both types invest. The detection equilibrium is similar except that the low type always reports high. Since only the low type is assumed to report low, the auditor always accepts a low report (that is, \(\alpha_l = 0\) and \(B_l^h\) = 1). Based on Observations 1 and 2 and the payoff specifications in Section 2.2, the objective functions in the detection and deterrence equilibrium are: The firm of type \(l\) chooses \(\rho(h|l)\) to maximize:

\[
\rho(h|l)((1 - B_h^l[\alpha_l])(v_l[V_L] - R - F) + B_h^l[\alpha_l](v_l[V_H] - F)) + (1 - \rho(h|l))(v_l[V_L] - F).
\]  

(A-1)

The firm of type \(h\) chooses \(\rho(h|h)\) to maximize:

\[
\rho(h|h)((1 - \alpha_h)(v_h[V_H] - F) + \alpha_h(w_h - R - F)) + (1 - \rho(h|h))(w_h - F).
\]  

(A-2)

The auditor examining report \(h\) chooses \(\alpha_h\) to maximize:

\[
p(h)\rho(h|h)[(1 - \alpha_h)(F - C_S) + \alpha_h(F - C_S - C_I)] \\
+ (1 - p(h))\rho(h|l)[(1 - B_h^l[\alpha_l])(F - C_S) + B_h^l[\alpha_l](F - C_S - C_I)].
\]  

(A-3)

The first line of (A-1) is the expected utility to the low type firm if it reports high. With probability \(1 - B_h^l[\alpha_l]\), the high report is rejected so the low type receives \(v_l[V_L]\) and pays an audit adjustment cost \(R\).
With probability $B_{hi} \alpha_{hi}$, the high report is accepted so the low type nets $v_l[V_H]$. If the low type reports low, the auditor finds it optimal to accept that report as truthful. Thus, the low type receives $v_l[V_L]$ without having to pay a rejection cost. Expression (A-2) is interpreted similarly. In expression (A-3), the auditor receives his fee net of sampling costs, $F - Cs$, in all cases. But if he rejects (accepts) the high report from the high (low) type, he pays an additional Type I error cost $C_I$ (Type II error cost $C_{II}$). Since both the detection and deterrence equilibria involve only the low type possibly reporting low, the auditor examining the unaudited low report always accepts it.

**Step 2:** The possible equilibrium reporting strategies are:

<table>
<thead>
<tr>
<th></th>
<th>Low type reports high</th>
<th>Low type randomizes between high and low reports</th>
<th>Low type reports low</th>
</tr>
</thead>
<tbody>
<tr>
<td>High type reports high</td>
<td>Class 1:</td>
<td>Class 2:</td>
<td>Class 3: Does not exist by Lemma 1.</td>
</tr>
<tr>
<td>High type randomizes between high and low reports</td>
<td>Class 4: Informationally equivalent to Class 6</td>
<td>Class 5: Informationally equivalent to Class 3</td>
<td>Class 6: Does not exist by Lemma 2</td>
</tr>
<tr>
<td>High type reports low</td>
<td>Class 7: Informationally equivalent to Class 3</td>
<td>Class 8: Informationally equivalent to Class 2</td>
<td>Class 9: Informationally equivalent to Class 1</td>
</tr>
</tbody>
</table>

Table 2: Classes of Equilibrium Reporting Strategies

Classes 4, 7, 8, and 9 are informationally equivalent to Classes 6, 3, 2, and 1 respectively because, for instance, the report $\hat{l}$ is not the literal statement “low” but rather it is a report consistent in equilibrium with the statement “I am a low type.” For instance, when the high type firm always reports $\hat{l}$ and the low type firm randomizes between $\hat{h}$ and $\hat{l}$, the auditor must in equilibrium interpret $\hat{l}$ to mean “I am a high type.” Thus, for example, equilibria in Class 6 wherein the high type firm randomizes between reports and the low type firm always reports low is informationally equivalent to equilibria in Class 4 wherein the high type firm randomizes between reports and the low type firm always reports high.

We show that the equilibria in Classes 3, 7, 6, and 4 do not exist. Then we show that equilibria in Class 5 do not exist generically. Then in the following steps we show that equilibrium with reporting strategies in Classes 1 or 2 exists and is unique.

**Lemma 1** Fully separating reporting (in Classes 3 and 7) does not occur in equilibrium.
Proof. Assume that the high type always reports high and the low type always report low. Then the auditor always accepts both the high report and low report because he suffers losses from incorrectly rejecting either report. Assume that investors set firm values assuming that each report is correct. But now, the low type defects to reporting high because his payoff to reporting high is $v_l[V_H^i] - F$, while his payoff from reporting low is $v_l[V_L^i] - F$.

Lemma 2  Equilibria in Class 6 and 4 do not exist.

Proof. Assume that the high type randomizes between reports and the low type always report low. Then the auditor always accepts the high report (i.e., $\alpha_h = 1$) because he suffers losses from incorrectly rejecting a report. By assumption the auditor must choose an equilibrium $\alpha_l$ that makes the high type indifferent between reports. Assume that investors set firm values believing that the high report is correct and that the low report could come from a high or low type. Then the high type’s payoff from reporting high is $v_h[V_H^i] - F$, while his payoff from reporting low is $(1 - B_l[\alpha_l])[v_h[V_H^i] - F - R] + B_l[\alpha_l](w_h - F)$. Clearly it is a dominant strategy for the high type to report high regardless of the $\alpha_l$ selected by the auditor. This contradicts the requirement that the auditor must choose an $\alpha_l$ that makes the high type indifferent between reporting high or low.

Lemma 3  Equilibria in Class 5 do not exist generically.

Proof. Assume that both the high type and the low type randomize between reports. Then the auditor must choose an equilibrium $\alpha_h$ and $\alpha_l$ that makes each type indifferent between reports. Assume that investors set firm values assuming that either report could come from either type. Then the high type’s payoff from reporting high must be equal to his payoff from reporting low. That is,

$$(1 - \alpha_h)[v_h[V_H^i] - F] + \alpha_h(w_h - R - F) = (1 - B_l[\alpha_l])[v_h[V_H^i] - F - R] + B_l[\alpha_l](w_h - F)$$

Recall that

$$v_h[V_H^i] = \frac{V_H^i - I}{V_H^i}(w_h + E[x] + I).$$
Recall that each of the three equations above, $f^V$, $g^V$, and $h^V$, are functions of the exogenous variables and the auditor’s two decision variables $h^V(\alpha_h, \alpha_l)$.

A third restriction on $V_H$ is that investors use Bayes rule to set the firm’s market value:

$$V_H = \frac{\rho(h)[(1 - \alpha_h) + \rho(l)B^V_h(\alpha_h)] + w_l(1 - p(h))[\rho(h)B^V_h(\alpha_h) + \rho(l)\alpha_l]}{E[x] + I} = h(\alpha_h, \alpha_l)$$

(A-4)

Each of the three equations above, $f(\alpha_h, \alpha_l)$, $g(\alpha_h, \alpha_l)$ and $h(\alpha_h, \alpha_l)$, are functions of the exogenous variables and the auditor’s two decision variables $h(\alpha_h, \alpha_l)$. Fixing the exogenous variables, the system of three equations in two unknowns is “overdetermined”. That is, except for “knife-edge” conditions on the exogenous variables, the system of equations does not have a solution. A more precise discussion of a non-generic equilibrium involves the following: Let the exogenous parameters of a given setting be a point \{w_h, w_l, E[x], I, R, C_1, C_{II}, C_S\} in the space $R^8$. Then an equilibrium is nongeneric if the set of exogenous parameters for which an equilibrium exists is a set of measure zero in the set of parameters.
Together Lemmata 1-3 show that the only reporting strategies that appear in equilibrium involve both types reporting high or the high type reporting high and the low type randomizing between high and low reports.

**Step 3:**

**Lemma 4** If the equilibrium involves both types submitting an unaudited high report, there exist the following auditor decision rules:

1. \( \alpha_h^* \in [0, 1] \), which is the auditor’s optimal decision rule.
2. \( \alpha_h^{LR^*} \in [0, 1] \) at which the low type is indifferent between unaudited reports and below which the low type prefers to report high.
3. \( \alpha_h^{HR^*} < 1 \) at which the high type is indifferent between unaudited reports and below which the high type prefers to report high.
4. \( \alpha_h^{HI^*} \) at which the high type with the audited high report is indifferent about investing and above which the high type prefers to invest.

If the equilibrium involves the high type submitting an unaudited high report and the low type randomizing between unaudited reports, there exists the following auditor decision rules:

5. \( \alpha_h^{**} \in [0, 1] \) at which the low type is indifferent between unaudited reports and below which the low type prefers to report high and which is the auditor’s best response to \( \rho(\hat{h}|l) \in [0, 1] \). \( \alpha_h^{HR^{**}} < 1 \) at which the high type is indifferent between unaudited reports and below which the high type prefers to report high.
6. \( \alpha_h^{HI^{**}} > 0 \) at which the high type with the audited high report is indifferent about investing and above which the high type prefers to invest.

**Proof.**

1. From the first order conditions of the auditor’s objective function in (A-3) when \( \rho(\hat{h}|l) = 1 \), the
decision rule that maximizes the auditor’s payoff is the $\alpha_h^*$ that solves

$$-B'_h[\alpha_h^*] = \frac{p(h)C_I}{(1 - p(h))C_{II}}. \tag{A-5}$$

The second order condition $(1 - p(h))C_{II}B''_h[\alpha_h^*] < 0$ is satisfied by Proposition 0(i). So $\alpha_h^*$ always exists in $[0, 1]$.

2. The low type is indifferent among reports in a detection equilibrium if

$$B_h[\alpha_h^{LR*}] = \frac{R}{v_l[V_H[\alpha_h^*, l]] - (w_l + E[x] + A) + R} \tag{A-6}$$

Substituting into (A-6) the expression for $v_l[V_H[\alpha_h^*, l]]$ and the expressions for $V_H[\alpha_h^*]$ from Remark 5b, where $\rho(\hat{h}|l) = 1$ results in a quadratic equation in $\beta_h$. Additional calculations show the positive solution to the quadratic equation that specifies $\beta_h$ as a function, denoted $k[\cdot]$, of $\alpha_h$ and other exogenous variables:

$$B_h = k[\alpha_h] = \frac{1}{2} \left[ \frac{(w_h + E[x] + I)R + I(w_h - w_l - A)}{2(w_l + E[x] + I + A)R} (1 - \alpha_h) \frac{p(h)}{1 - p(h)} \right]$$

$$+ \left\{ \left[ \frac{(w_h + E[x] + I)R + I(w_h - w_l)}{2(w_l + E[x] + I + A)R} (1 - \alpha_h) \frac{p(h)}{1 - p(h)} \right]^2 \right.$$  

$$\left. + \frac{w_h + E[x] + I}{w_l + E[x] + I + A} (1 - \alpha_h) \frac{p(h)}{1 - p(h)} \right\}^{1/2}.$$  

$k[\cdot]$ is the locus of auditor tests that makes the low type firm indifferent between reporting high and low taking into account the investors’ endogenously derived market value in the detection equilibrium. Tediou calculations show, that $k[\cdot]$ is always strictly greater than zero, strictly less than one, and strictly increasing in $\alpha$. Since $B_h[\cdot]$ is always strictly between zero and one and is strictly decreasing in $\alpha$, there exists an $\alpha_h^{LR*}$ that solves $B_h[\alpha_h^*] = k[\alpha_h]$.

For any $\alpha_h < (> \, \alpha_h^{LR*}$ the low type firm strictly prefers to report high (low).

3. From equation (A-2), the high type always reports high, if

$$\alpha_h^* \leq \frac{v_l[V_H[\alpha_h^*]] - w_h}{v_l[V_H[\alpha_h^*]] - w_h - R} \equiv \alpha_h^{HR*} \tag{A-7}$$
By construction, $0 < \alpha^{HR*}_h < 1$. In a detection equilibrium in which $\alpha^{*}_h \in [0, \alpha^{HR*}_h]$, the high type would always report high, while if $\alpha^{*}_h \in (\alpha^{HR*}_h, 1]$, the high type would defect to reporting low.

4. The high type with the audited high report invests in a detection equilibrium if and only if

$$v_h[V_H] = \frac{V_H[\alpha^*_h] - I}{V_H[\alpha^*_h]}(w_h + E[x] + I) \geq w_h.$$ 

Rearranging yields

$$V_H[\alpha^*_h] = \frac{w_hp(h)(1 - \alpha^*_h) + w_l(1-p(h))(B_h[\alpha^*_h])}{p(h)(1 - \alpha^*_h) + (1-p(h))(B_h[\alpha^*_h])} + E[x] + I \geq \frac{(w_h + E[x] + I)I}{E[x] + I},$$

or

$$\frac{(V_h - \frac{V_hI}{E[x] + I})p(h)}{(V_hI - V_l)(1 - p(h))} (1 - \alpha^*_h) \geq B_h[\alpha^*_h],$$

where $V_l = w_l + E[x] + I$, $t \in \{h, l\}$. The coefficient on $(1 - \alpha^*_h)$ is positive because the numerator is positive by inspection and the denominator is also positive because, 

$$\frac{V_hI}{E[x] + I} - V_l = \frac{w_hI}{E[x] + I} + I - V_l$$

$$= \left( \frac{w_hI}{E[x] + I} - w_l - E[x] \right)$$

$$> \left( \frac{w_hI}{E[x] + I} - E[w] - E[x] \right) > 0.$$ 

The coefficient on $(1-\alpha^*_h)$ is less than one because

$$\frac{(V_h - \frac{V_hI}{E[x] + I})p(h)}{(V_hI - V_l)(1 - p(h))} < 1$$

if and only if $\left( V_h - \frac{V_hI}{E[x] + I} \right)p(h) < \left( \frac{V_hI}{E[x] + I} - V_l \right)(1 - p(h))$, if and only if $V_{\phi} < \frac{V_hI}{E[x] + I}$, which follows from assumption (2).

There exists a unique $\alpha^{HR*}_h \in (0, 1]$ that solves $\frac{(V_h - \frac{V_hI}{E[x] + I})p(h)}{(V_hI - V_l)(1 - p(h))} (1 - \alpha^*_h) = B_h[\alpha^*_h]$. For all $\alpha^*_h \in [\alpha^{HR*}_h, 1]$, the high type with the audited high report would be willing to invest in a detection equilibrium.
5. The low type is indifferent between reports in a deterrence equilibrium if

\[ B_h[\alpha_h^{**}] = \frac{R}{\nu_l[V_H[\alpha_h^{**}, \rho(\hat{h}|l)]] - (w_l + E[x] + A) + R} \]  \hspace{1cm} (A-8)

Substituting into (A-8) the expression for \( \nu_l[V_H[\alpha_h^{**}, \rho(\hat{h}|l)]] \) and the expressions for \( V_H[\alpha_h^{**}, \rho(\hat{h}|l)] \) from Remark 5c and the expression for \( \rho(\hat{h}|l) \) from (A-4) results in a quadratic equation in \( B_h \). Once again additional calculations show the positive solution to the quadratic equation that specifies \( B_h \) as a function, denoted \( m[\cdot] \), of \( \alpha_h, B_h[\cdot] \), and other exogenous variables:

\[ B_h = m[\alpha_h] = \frac{1}{2} - \frac{(w_h + E[x] + I)R + I(w_h - w_l - A)}{2(w_l + E[x] + I + A)R} \left( 1 - \alpha_h \right) \frac{C_{II}}{C_I} \] \hspace{1cm} (A-9)

\[ + \left\{ \left[ \frac{(w_h + E[x] + I)R + I(w_h - w_l - A)}{2(w_l + E[x] + I + A)R} \left( 1 - \alpha_h \right) \frac{C_{II}}{C_I} - \frac{1}{2} \right]^2 \right. \]

\[ + \left. \frac{(w_h + E[x] + I)}{(w_l + E[x] + I + A)} \left( 1 - \alpha_h \right) \frac{C_{II}}{C_I} \right\}^{\frac{1}{2}} \]

\( m[\cdot] \) is the locus of auditor tests that makes the low type firm indifferent between reports taking into account the investors’ endogenously derived market value in the deterrence equilibrium. Further calculations show, that \( m[\cdot] \) is always strictly greater than zero, strictly less than one, and strictly increasing in \( \alpha \). Since \( B_h[\cdot] \) is always strictly between zero and one and is strictly decreasing in \( \alpha \), there exists an \( \alpha_h^{**} \) that solves \( B_h[\alpha_h] = m[\alpha_h] \). For any \( \alpha_h < (>) \alpha_h^{**} \) the low type firm strictly prefers to report high (low).

From the first order conditions of the auditor’s objective function in (A-3) when \( \rho(\hat{h}|l) < 1 \), the auditor has optimized at \( \alpha_h^{**} \) if \( \rho(\hat{h}|l) \) is such that

\[ -B'_h[\alpha_h^{**}] = \frac{p(h)C_I}{(1 - p(h))\rho(\hat{h}|l)C_{II}}. \]  \hspace{1cm} (A-10)

That is,

\[ \rho(\hat{h}|l) = \frac{p(h)C_I}{(1 - p(h))C_{II}(-B'_h[\alpha_h^{**}])}. \]  \hspace{1cm} (A-11)
The second order condition $(1 - p(h))\rho(\hat{h}|l)C_{II}B_{h}^{\alpha_{h}^{**}} < 0$ is satisfied by Proposition 0(i). Note that the low type firm’s randomizing strategy must be feasible, i.e., $\rho(\hat{h}|l) \in (0, 1)$. We check this condition in Lemma 5.

6. From (A-2), the high type always reports high in the deterrence equilibrium if

$$\alpha_{h}^{**} \leq \frac{v_{h}[V_{h}^{*}[\alpha_{h}^{**}, \rho(\hat{h}|l)]] - w_{h}}{v_{h}[V_{h}] - w_{h} + R} \equiv \alpha_{h}^{HR^{**}}$$

(A-12)

By construction, $0 < \alpha_{h}^{HR^{**}} < 1$. In a deterrence equilibrium in which $\alpha_{h}^{**} \in [0, \alpha_{h}^{HR^{**}}]$, the high type would always report high, while if $\alpha_{h}^{**} \in (\alpha_{h}^{HR^{**}}, 1]$, the high type would defect to reporting low.

7. The high type invests in a deterrence equilibrium if and only if

$$v_{h}[V_{h}] = \frac{v_{h}[V_{h}^{*}[\alpha_{h}^{**}, \rho(\hat{h}|l)]] - I}{v_{h}[V_{h}^{*}[\alpha_{h}^{**}, \rho(\hat{h}|l)]]} = (w_{h} + E[x] + I) \geq w_{h}.$$

Rearranging yields

$$V_{h}^{*}[\alpha_{h}^{**}, \rho(\hat{h}|l)] = \frac{w_{h}p(h)(1 - \alpha_{h}) + w_{l}(1 - p(h))\rho(\hat{h}|l)(B_{h}[\alpha_{h}^{**}])}{p(h)(1 - \alpha_{h}) + (1 - p(h))\rho(\hat{h}|l)(B_{h}[\alpha_{h}^{**}])} + E[x] + I \geq \frac{(w_{h} + E[x] + I)I}{E[x] + I},$$

or

$$\left(\frac{V_{h} - \frac{V_{h}I}{E[x]+I}}{V_{h}}\right) p(h) \left(\frac{V_{h}I}{E[x]+I} - V_{l}\right) (1 - p(h))\rho(\hat{h}|l) (1 - \alpha_{h}) > B_{h}[\alpha_{h}^{**}].$$

Similar to Lemma 4(4), the coefficient on $(1 - \alpha_{h})$ is positive.

Case 1: If

$$0 < \frac{\left(\frac{V_{h} - \frac{V_{h}I}{E[x]+I}}{V_{h}}\right) p(h)}{\left(\frac{V_{h}I}{E[x]+I} - V_{l}\right) (1 - p(h))\rho(\hat{h}|l)} < 1,$$
then there exists an $\alpha_h^{HR^*} \in (0,1]$ that solves

$$\left( \frac{V_h - \frac{V_h L}{E[x]} - V_I}{E[x] - V_I} \right) p(h) (1 - \alpha_h^*) = B_{h}[\alpha_h^*].$$

For all $\alpha_h^* \in [\alpha_h^{HR^*}, 1]$, the high type would be willing to invest in a deterrence equilibrium.

Case 2: If

$$\left( \frac{V_h - \frac{V_h L}{E[x]} - V_I}{E[x] - V_I} \right) p(h) (1 - \alpha_h^*) = 1,$$

then $\alpha_h^*=0$ and for every $\alpha_h^* \in (0,1]$, the high type would be willing to invest in equilibrium. Now we state the necessary and sufficient conditions for existence of equilibrium in terms of the decision rules identified in Lemma 4:

**Remark 6** The necessary and sufficient conditions for a detection equilibrium to exist are:

(P1) The auditor’s decision rule given the unaudited high report maximizes his payoff. That is, the auditor’s equilibrium decision rule is $\alpha_h^*$ and it exists in the interval $[0,1]$.

(P2) The auditor’s decision rule given the unaudited low report maximizes his payoff. Since the auditor believes that an out-of-equilibrium unaudited low report comes from the low type, the auditor finds it optimal to always accept that report, i.e., $\alpha_l^*=0$.

(P3) The low type always reports high, i.e., $\alpha_h^* < \alpha_h^{LR^*}$.

(P4) The high type always reports high, i.e., $\alpha_h^* < \alpha_h^{HR^*}$.

(P5) The high type with the audited high report invests, i.e., $\alpha_h^{HR^*} < \alpha_h^*$.

(P6) The low type invests. This condition is always satisfied.

The necessary and sufficient conditions in the deterrence equilibrium to exist are:

(SS1) The auditor’s decision rule given the high report maximizes his payoff. That is, the auditor’s equilibrium decision rule is $\alpha_h^{**}$ and it exists in the interval $[0,1]$.

(SS2) The auditor’s decision rule given the unaudited low report maximizes his payoff. Since the
unaudited low report is released only by the low type, the auditor finds it optimal to always accept that report, i.e., $\alpha_{l}^{**} = 0$.

(SS3) The low type is indifferent between reports, i.e., $\alpha_{h} = \alpha_{h}^{**}$, and his randomizing strategy is feasible, i.e., $\rho(\hat{h}|l) \in (0, 1)$.

(SS4) The high type always reports high, i.e., $\alpha_{h}^{**} < \alpha_{h}^{HR^{**}}$.

(SS5) The high type with the audited high report invests, i.e., $\alpha_{h}^{H_{I}^{**}} < \alpha_{h}^{**}$.

(SS6) The low type invests. This condition is always satisfied by Lemma4(i).

Lemma 5 Three possibilities exist

1. $\alpha_{h}^{H_{I}} < \alpha_{h}^{*} < \alpha_{h}^{**}$ in which case the detection equilibrium prevails and the auditor’s decision rule is $\alpha_{h}^{*}$.

2. $\alpha_{h}^{H_{I}} < \alpha_{h}^{*} < \alpha_{h}^{**}$ in which case the deterrence equilibrium prevails and the auditor’s decision rule is $\alpha_{h}^{**}$.

3. Otherwise the ineffective-auditing equilibrium prevails.

Proof. Note from the definitions of $\alpha_{h}^{**}$ and $\alpha_{h}^{*}$ in (A-12) and (A-7), respectively, if the audit adjustment cost $R$ is small enough relative to the difference between the market price and the no-investment value of each type of firm, $\alpha_{h}^{HR^{**}}$ and $\alpha_{h}^{HR^{*}}$ are both close to one and conditions (SS4) and (P4) are satisfied.

Equilibrium would exist and is unique for any values of $R$. However, when $R$ is small relative to the gains to each type from receiving the audited high report, the full range of possible equilibria exist. Furthermore, a relatively small $R$ is empirically descriptive. If $R$ were zero, the low type would always report high for it is costless to do so. In this case, only detection and ineffective-auditing equilibria would exist.

In the model, if $R$ were too large, then the high type would not report high in the detection or deterrence equilibrium. He would rather have the unaudited low report be accepted with certainty than to suffer,
with some (possibly small) probability, a very large penalty for a rejected high report. Consequently, only ineffective-auditing equilibria would exist.

Assuming the high type invests, there are two possibilities: \( \rho(\hat{h}|l) = 1 \)

Case 1: \( \alpha^{**}_h > \alpha^*_h \) if and only if, by Proposition 0 (iv), \(-B'_h[\alpha^{**}_h] < -B'_h[\alpha^*_h] \). By (A-5) and (A-10), this implies \( \frac{p(h)C_I}{(1-p(h))C_{II}p(h|l)} < \frac{p(h)C_I}{(1-p(h)) C_{II}} \), or equivalently that \( \rho(\hat{h}|l) > 1 \). That is, the firm’s randomizing strategy is not feasible and condition (SS3) is not satisfied. That is, if \( \alpha^{**}_h > \alpha^*_h \), the deterrence equilibrium cannot exist.

Also \( \rho(\hat{h}|l) > 1 \) implies \( (-B'_h[\alpha^{**}_h])_{C_{II}} < \frac{p(h)C_I}{1-p(h)} \). Since \(-B'_h[\alpha_h] \) is decreasing in \( \alpha_h \), \(-B'_h[\alpha^{**}_h] \) is decreasing in \( \alpha_h \) and \(-B'_h[\alpha^*_h] \) is decreasing in \( \alpha^*_h \). Thus, \( \alpha^{LR*}_h > \alpha^{**}_h \) which is defined as the solution of \( B_h[\alpha_h] = \frac{C_{II}}{C_{II}} \) for every \( \alpha_h \in [\alpha^{**}_h, 1] \). Therefore, \( k[\alpha_h] < m[\alpha_h] \) for every \( \alpha_h \in [\alpha^{**}_h, 1] \). Note that \( k[\cdot] \) and \( m[\cdot] \) are strictly increasing on \([0,1]\) and \( B_h[\alpha_h] \) is strictly decreasing on \([0,1]\). Thus, \( \alpha^{LR*}_h > \alpha^{**}_h \) and \( \alpha^{**}_h > \alpha^*_h \) yields \( \alpha^{LR*}_h > \alpha^*_h \). That is, if \( \alpha^{**}_h > \alpha^*_h \), condition (P3) is satisfied and the detection equilibrium exists.

Case 2: \( \alpha^{**}_h < \alpha^*_h \) if and only if, by Proposition 0 (iv), \(-B'_h[\alpha^{**}_h] > -B'_h[\alpha^*_h] \). By (A-5) and (A-10), this implies \( \frac{p(h)C_I}{(1-p(h))C_{II}p(h|l)} > \frac{p(h)C_I}{(1-p(h)) C_{II}} \), or equivalently that \( \rho(\hat{h}|l) \in (0,1) \). That is, the firm’s randomizing strategy is feasible and condition (SS3) is not satisfied.

Furthermore, \( \rho(\hat{h}|l) < 1 \) implies \( (-B'_h[\alpha^{**}_h])_{C_{II}} > \frac{p(h)C_I}{1-p(h)} \). Since \(-B'_h[\alpha_h] \) is decreasing in \( \alpha_h \), \(-B'_h[\alpha^{**}_h] \) is decreasing in \( \alpha_h \) and \(-B'_h[\alpha^*_h] \) is decreasing in \( \alpha^*_h \). Note that \( k[\cdot] \) and \( m[\cdot] \) are strictly increasing on \([0,1]\) and \( B_h[\alpha_h] \) is strictly decreasing on \([0,1]\). Thus, \( \alpha^{LR*}_h > \alpha^{**}_h \) which is defined as the solution of \( B_h[\alpha_h] = m[\alpha_h] \) must be greater than \( \alpha^{**}_h \) defined as the solution to \( B_h[\alpha_h] = m[\alpha_h] \). Combining \( \alpha^{LR*}_h > \alpha^{**}_h \) and \( \alpha^{**}_h > \alpha^*_h \) yields \( \alpha^{LR*}_h > \alpha^*_h \). That is, if \( \alpha^{**}_h > \alpha^*_h \), condition (P3) is satisfied and the detection equilibrium exists.

So far we have assumed that the high type invests, that is, that (P5) or (SS5) is satisfied. We add this condition to the cases 1 and 2 to derive the conditions for existence and uniqueness in Lemma 5(i) and 5(ii). If these conditions are violated, the high type does not invest and the ineffective-auditing equilibrium prevails.

**Step 4:** To demonstrate existence, we provide a numerical example of each type of equilibrium as well as the equilibrium at the optimal level of auditor liability.
Consider the example in section 2.2 of Myers and Majluf (1984). They assume that \( w_h = 150 \), \( w_l = 50 \), \( I = 100 \) and \( p(h) = 0.5 \). Unlike MM, we assume symmetric uncertainty regarding the expected net present value of the new project, thus for each type \( E[x] = 15 \) and the return on the project is 15%. To highlight the effect of the cost of a type II error on Myers and Majluf (1984), we further assume that the rejection cost \( R = 1 \), the Type I error costs \( C_I = 1 \), and the damage award \( A = 0 \). Assume that the audit technology is \( B_h[\alpha_h] = [1 - (\alpha_h)^{0.5}]^2 \). Since \( B_h[\alpha_h] \) is everywhere convex and strictly decreasing on \( B_h[\alpha_h] \) by Proposition 0 (vii), this is an admissible audit technology.

Note that \( V_0 = 215 \), \( v_h[V_0] = \frac{115}{215} (265) = 141.74 < w_h = 150 \), thus the high type does not invest in a setting with no auditing. Note that market value with the audited high report needs to be above 230 so that \( v_h[230] = \frac{130}{230} (265) = 150 \).

Note that in the ineffective-auditing equilibrium only the low type invests and thus investors assign the full information low value \( V_l \) to firms with audited high and low reports. There are no audit failures because the firm ex post turned out to be exactly the expected type. Thus, the equilibrium in the lemons setting with auditing always exists, is unique and is either an ineffective-auditing equilibrium, a detection equilibrium, or a deterrence equilibrium. This concludes the proof of Proposition 1. Q.E.D.

**Proof of Proposition 2:** First we prove that \( \frac{\partial \alpha_h^*}{\partial C_{II}} > 0 \). Implicit differentiation yields

\[
\frac{\partial \alpha_h^*}{\partial C_{II}} = \frac{p(h)B_h[\alpha_h]}{(1 - p(h))C_{II}^2} > 0.
\]

Second, we need to show that \( \frac{\partial \alpha_h^{**}}{\partial C_{II}} > 0 \). Take the derivative of \( m[\alpha_h] \) from Lemma 4(v) and simplify it by simple algebra. It can be shown that the sign of \( \frac{\partial \alpha_h^{**}}{\partial C_{II}} \) is the sign of

\[
4 \left( \frac{C_{II}}{C_{II}^2} \right)^2 \frac{(w_h + E[x] + I)R + I(w_h - w_l - A)}{(w_h + E[x] + I)R + I(w_h - w_l - A)^2} > 0.
\]

In particular, the \( \alpha_h^{**} \) that solves \( B_h[\alpha_h] = m[\alpha_h] \) decreases in \( \frac{C_{II}}{C_{II}^2} \), thus increases in \( C_{II} \) or \( \frac{\partial \alpha_h^{**}}{\partial C_{II}} > 0 \). Third, returning to the proof of Proposition 2. The probability of new investment in the detection equilibrium are
Table 3: Numerical Example

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Auditor liability level $C_{II}$</td>
<td>0.25</td>
<td>3.00</td>
<td>8.00</td>
</tr>
<tr>
<td>Auditor decision given $\alpha_{\hat{h}}$</td>
<td>0.04</td>
<td>0.56</td>
<td>0.70</td>
</tr>
<tr>
<td>Misreporting rate $\rho(h</td>
<td>l)$</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Investors’ valuation with audited high report, $V_{\hat{H}}[\alpha_{\hat{h}}, \rho(\hat{h}</td>
<td>l)]$</td>
<td>165.00</td>
<td>252.5</td>
</tr>
<tr>
<td>Retain firm value of high firm with high report $v_{h}[V_{\hat{H}}]$</td>
<td>150.00</td>
<td>160.05</td>
<td>162.93</td>
</tr>
<tr>
<td>Expected value to current shareholders</td>
<td>107.50</td>
<td>110.78</td>
<td>109.75</td>
</tr>
<tr>
<td>Retain firm value of high firm with high report $v_{h}[V_{\hat{H}}]$</td>
<td>0.0%</td>
<td>3.1%</td>
<td>0.9%</td>
</tr>
<tr>
<td>Probability of litigation related to an audit failure $(1 - p(h))\rho(\hat{h}</td>
<td>l) B_{\hat{h}}[\alpha_{\hat{h}}]$</td>
<td>60.6%</td>
<td>4.95%</td>
</tr>
<tr>
<td>Cost of capital of high firm audit failure w/ audited high report</td>
<td>60.6%</td>
<td>60.6%</td>
<td>60.6%</td>
</tr>
<tr>
<td>Cost of capital of low firm audit failure w/ audited low report</td>
<td>0.00%</td>
<td>-34.7%</td>
<td>-36.4%</td>
</tr>
<tr>
<td>Cost of capital of low firm audit failure w/ audited low report</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Probability of new investment $(p(h)(1 - \alpha_{\hat{h}}) + (1 - p(h))$</td>
<td>50%</td>
<td>71.9%</td>
<td>65%</td>
</tr>
</tbody>
</table>
denoted $G^* = p(h)(1 - \alpha^*_h)$. It is straightforward to show that

$$\frac{\partial G^*}{\partial C_{II}} = -p(h) \frac{\partial \alpha^*_h}{\partial C_{II}} < 0.$$ 

Similarly, the probability of new investment in the detection equilibrium are denoted $G^{**} = p(h)(1 - \alpha^{**}_h)$. And

$$\frac{\partial G^{**}}{\partial C_{II}} = -p(h) \frac{\partial \alpha^{**}_h}{\partial C_{II}} < 0.$$ 

Q.E.D.

**Proof of Proposition 3:** Let the audit failure rate in the detection equilibrium be denoted $A F R^* = (1 - p(h))B^*_h[\alpha^*_h]$. Then

$$\frac{\partial A F R^*}{\partial C_{II}} = (1 - p(h))B^*_h[\alpha^*_h] \frac{\partial \alpha^*_h}{\partial C_{II}} < 0,$$

by Proposition 0(i) and Proposition 2. Similarly, let the audit failure rate in the deterrence equilibrium be denoted $A F R^{**} = (1 - p(h))\rho(\hat{h}|l)B^*_h[\alpha^{**}_h]$. Since

$$\rho(\hat{h}|l) = \frac{C_I}{C_{II}(1 - p(h))(-B^*_h[\alpha^*_h])},$$

$$A F R^{**} = (1 - p(h))\rho(\hat{h}|l)B^*_h[\alpha^{**}_h].$$

Then

$$\frac{\partial A F R^{**}}{\partial C_{II}} = \frac{C_I}{C_{II}} p(h) \left[ \frac{-B^*_h[\alpha^{**}_h]}{C_{II}(-B^*_h[\alpha^*_h])} - \frac{\partial \alpha^{**}_h}{\partial C_{II}} \right].$$

When the audit technology is discrete, the sign of the derivative is negative by Proposition 2 and Proposition 0(i). Q.E.D.

**Proof of Proposition 4:** Denote the cost of capital to the high type with the high report in the detection equilibrium as

$$k_h(\hat{H}^*) = \frac{w_h - E[w|\hat{H}^*]}{V_{\hat{H}^*}}.$$
Then
\[
\frac{\partial k_h(\hat{H}^*)}{\partial C_{II}} = \frac{p(h)(1 - p(h))(w_h - w_l - A)V_h \left(B_h[\alpha_h^*] + (1 - \alpha_h^*) \right) \frac{\partial \alpha_h^*}{\partial C_{II}}}{V_{\hat{H}^*}}
\]

By Proposition 0(vi) and Proposition 2, the sign of the derivative is negative in the detection equilibria. Similarly, \( k_l(\hat{H}^*) < 0 \). Denote the cost of capital to the high type with the high report in the deterrence equilibrium as
\[
k_h(\hat{H}^{**}) = \frac{w_h - E[w|\hat{H}^{**}]}{V_{\hat{H}^{**}}}.
\]

Then
\[
\frac{\partial k_h(\hat{H}^{**})}{\partial C_{II}} = \left(\frac{w_h - w_l - A}{V_{\hat{H}^{**}}} \right)^2 \left\{ 1 + C_{II} \frac{\partial \alpha_h^{**}}{\partial C_{II}} \left[ -\frac{B_h^l[\alpha_h^*]}{B_h[\alpha_h^*]} - \frac{1}{1 - \alpha_h^{**}} \right] \right\},
\]
when the audit technology is discrete. By Proposition 0(vi) and Proposition 2, the sign of the derivative is negative. Similarly, \( k_l(\hat{H}^{**}) < 0 \). Q.E.D.
Appendix 2: Notation

Types, unaudited reports, and audited reports

- $w_t$: value of the existing equity in firm type $t$
- $E[x]$: expected net present value of the investment project
- $I$: investment required to undertake the project
- $p(h)$: probability of high type firm
- $t \in \{h, l\}$: firm type, where $w_h > w_l$
- ${\hat{t}} \in \{{\hat{h}}, \hat{l}\}$: unaudited report from firm to auditor
- $\hat{T} \in \hat{H}, \hat{L}$: audited report from auditor to investors

The Firm’s Market Value

- $V_\phi = p(h)w_h + (1 - p(h))w_l + E[x] + I$: firm’s market value conditional on no information
- $V_T[\alpha_h, \rho(h|l)], \hat{T} = \hat{H}, \hat{L}$: firm’s market value conditional on audit report $\hat{T}$

Share of Firm Retained by Existing Shareholders

- $\frac{V_\phi - I}{V_\phi}$: share of firm retained by existing shareholders given no information and both types invest
- $\frac{V_{\hat{T}} - I}{V_{\hat{T}}}$: share of firm retained by existing shareholders given audit report $\hat{T}$ and both types invest

Retained Firm value

- $v_t[V_\phi] = \frac{V_\phi - I}{V_\phi} (w_t + E[x] + I)$: retained firm value of firm type $t$ given no information and both types invest
- $v_t[V_{\hat{T}}] = \frac{V_{\hat{T}} - I}{V_{\hat{T}}} (w_t + E[x] + I)$: retained firm value of firm type $t$ given audit report $\hat{T}$ and both types invest

Firm’s Strategies

- $\rho(\hat{t}|t)$: probability of reporting $\hat{t} \in \{{\hat{h}}, \hat{l}\}$ when $t \in \{h, l\}$

Auditor’s Payoffs

- $F$: audit fee
$C_S$ cost of sampling for auditor

$C_{II}$ cost of a Type II error, including a damage award $A$ and a legal cost $L$

$C_I$ cost of a Type I error

**Audit Technology**

$\omega \in \Omega$ audit evidence

$\{p(\omega|t), t \in \{h, l\}\}$ audit technology

**Auditor’s Strategies**

$\delta(d|\hat{t}, \omega)$ probability of making decision $d \in \{a, r\}$ given the null is $\hat{t}$ and the audit evidence is $\omega$

$\alpha_h^*$ probability, under decision rule $\delta$, of falsely rejecting the null of $\hat{t} = \hat{h}$ when the true value is $t = h$

$\beta_h^*$ probability, under decision rule $\delta$, of falsely accepting the null of $\hat{t} = \hat{h}$ when the true value is $t = l$

$\alpha_h^{**}$ Type I error prob. where low type indifferent between reports

$\alpha_h^{HR^{**}}$ Type I error prob. below which high type willing to report high in the deterrence equilibrium

$\alpha_h^{HI^{**}}$ Type I error prob. above which high type invests in the deterrence equilibrium

$\alpha_h^*$ Type I error prob. that is optimal for the auditor if both types report high

$\alpha_h^{HR^*}$ Type I error prob. below which the high type is willing to report high in the detection equilibrium

$\alpha_h^{LR^{**}}$ Type I error prob. below which the low type is willing to report high in the detection equilibrium

$\alpha_h^{HI^*}$ Type I error prob. above which the high type invests in the detection equilibrium
References


